

Ungleichungen

$$|8| = |-8| = 8$$

$$|x^2 - 2| \geq x$$

$$|x - 5| = 2 \quad \mathbb{L} = \{3; 7\}$$

a) Beispiel zur Fallunterscheidung

$$|x - 5| = 2x - 11$$

1. Fall

$$(x - 5) \geq 0$$

$$x - 5 = 2x - 11 \quad | -x + 11$$

$$6 = x$$

$$x = 6$$

$$|6 - 5| \stackrel{!}{\geq} 0$$

$$1 \geq 0 \quad \checkmark$$

$$\text{für } (x - 5) \geq 0 \rightarrow L_1 = \{6\}$$

$$L_1 \cup L_2 = \{6\}$$

2. Fall

$$(x - 5) < 0$$

$$-(x - 5) = 2x - 11$$

$$-x + 5 = 2x - 11 \quad | +x + 11$$

$$16 = 3x \quad | :3$$

$$x = \frac{16}{3}$$

$$\frac{16}{3} - 5 \stackrel{!}{<} 0$$

$$5,333 - 5 \stackrel{!}{<} 0$$

$$0,333 \stackrel{!}{<} 0$$

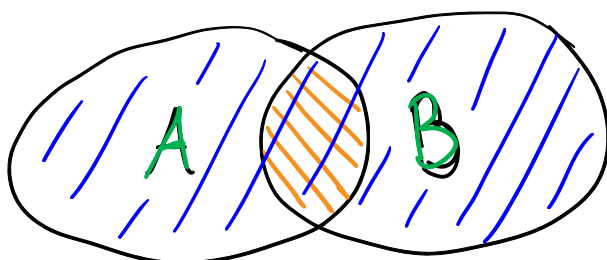
$$L_2 = \{\}$$

Vereinigungsmenge

$$A \cup B = \{1, 2, 3, 4, 5, 6, 8\}$$

Schnittmenge

$$A \cap B = \{2, 4, 6\}$$



$$A = \{1, 2, 3, 4, 5, 6\}$$

$$B = \{2, 4, 6, 8\}$$

$$b) |x^2 - 2| \geq x$$

1. Fall:

$$x^2 - 2 \geq 0:$$

$$x^2 - 2 \geq x \quad | -x$$

$$x^2 - x - 2 \geq 0$$

$$x_{1/2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{1 \pm \sqrt{1 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1} = \frac{1 \pm \sqrt{9}}{2} =$$

$$x_1 = \frac{1+3}{2} = 2 \quad x_2 = \frac{-2}{2} = -1$$

$$L_1 = \{-1, 2\}$$

$$L_1 \stackrel{!}{\geq} 0 \rightarrow \cancel{L_1 = \{-1, 2\}}$$

$$x^2 - 2 \stackrel{!}{\geq} 0 \quad \checkmark \quad L_1: x \geq 2 \rightarrow$$

2. Fall:

$$x^2 - 2 < 0:$$

$$-(x^2 - 2) \geq x$$

$$-x^2 + 2 \geq x \quad | -x$$

$$-x^2 - x + 2 \geq 0$$

$$x_{1/2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{1 \pm \sqrt{1 - 4 \cdot (-1) \cdot 2}}{2 \cdot (-1)}$$

$$= \frac{1 \pm \sqrt{9}}{-2} = \frac{1 \pm 3}{-2}$$

$$x_1 = \frac{4}{-2} = -2 \quad x_2 = \frac{-2}{-2} = 1$$

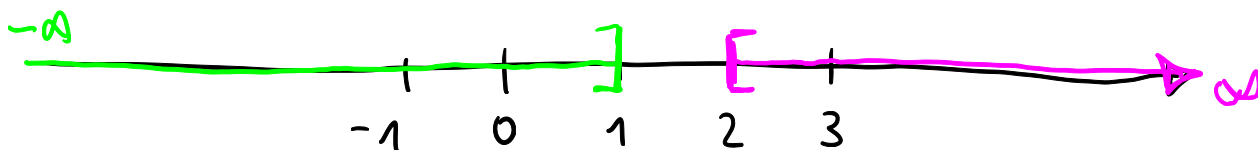
$$L_2 = \{-2, 1\}, \text{ aber } L_2 \stackrel{!}{\geq} 0$$

$$\cancel{L_2 = \{-2, 1\}} \rightarrow L_2: x \leq 1$$

$$x^2 - 2 \stackrel{!}{<} 0 \quad 1^2 - 2 \stackrel{!}{<} 0 \quad \checkmark$$

$$L = L_1 \cup L_2 =]-\infty; 1] \cup [2; \infty[$$

$$L =]-\infty; 1] \cup [2; \infty[= \mathbb{R} \setminus]1; 2[$$



$$c) \quad 3x + 5 = -2|x - 1|$$

1. Fall

$$x - 1 \geq 0$$

$$|x - 1| = x - 1$$

$$3x + 5 = -2(x - 1)$$

$$3x + 5 = -2x + 2 \quad | +2x - 5$$

$$5x = -3 \quad | :5$$

$$x = -\frac{3}{5}$$

Überprüfen, ob Bedingung erfüllt:

$$-\frac{3}{5} - 1 = -\frac{8}{5} \neq 0 \quad \text{⚡}$$

$$L_1 = \{ \}$$

2. Fall

$$x - 1 < 0$$

$$|x - 1| = -(x - 1)$$

$$3x + 5 = -2(-(x - 1))$$

$$3x + 5 = 2(x - 1)$$

$$3x + 5 = 2x - 2 \quad | -2x - 5$$

$$x = -7$$

Überprüfen:

$$-7 - 1 = -8 < 0 \quad \checkmark$$

$$L_2 = \{-7\}$$

$$L = L_1 \cup L_2 = \{-7\}$$

