Einführung

Quantum Computing

HSK 18.12.2024

- **QPE**
  - **Eigenvektoren**
  - **Quantumcircuit**
  - **Beispiel**

Eigenvektor

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$|x\rangle = \frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle$$

$$y = X|x\rangle = \frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$$

Eigenvektor

$$X\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{pmatrix} = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{pmatrix} = \begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{pmatrix}$$

$$X\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{-1}{\sqrt{2}}\end{pmatrix} = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}\frac{-1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{pmatrix} = -\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{-1}{\sqrt{2}}\end{pmatrix}$$

$$X|+\rangle = |+\rangle$$

$$X|-\rangle = -|-\rangle$$

Eigenvektor

$U\ |x> = \lambda\ |x>$

$\lambda = e^{i\,\alpha}$

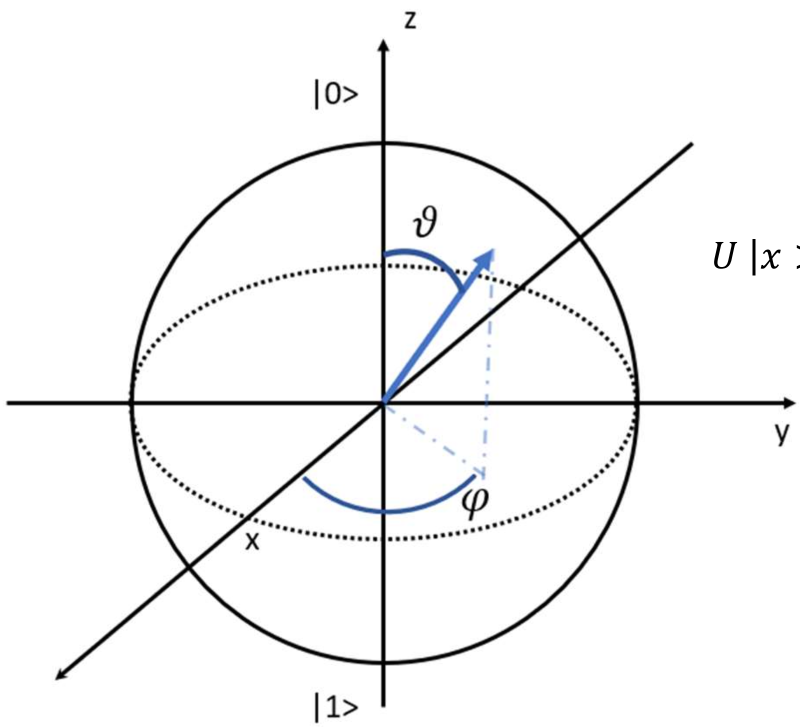
$$|x\rangle = \cos\left({}^{\vartheta}/_{2}\right)\begin{pmatrix}1\\0\end{pmatrix} + e^{i\varphi}\sin\left({}^{\vartheta}/_{2}\right)\begin{pmatrix}0\\1\end{pmatrix}$$

$\vartheta \in [0, \pi]$

$\varphi \in [0,2\pi]$

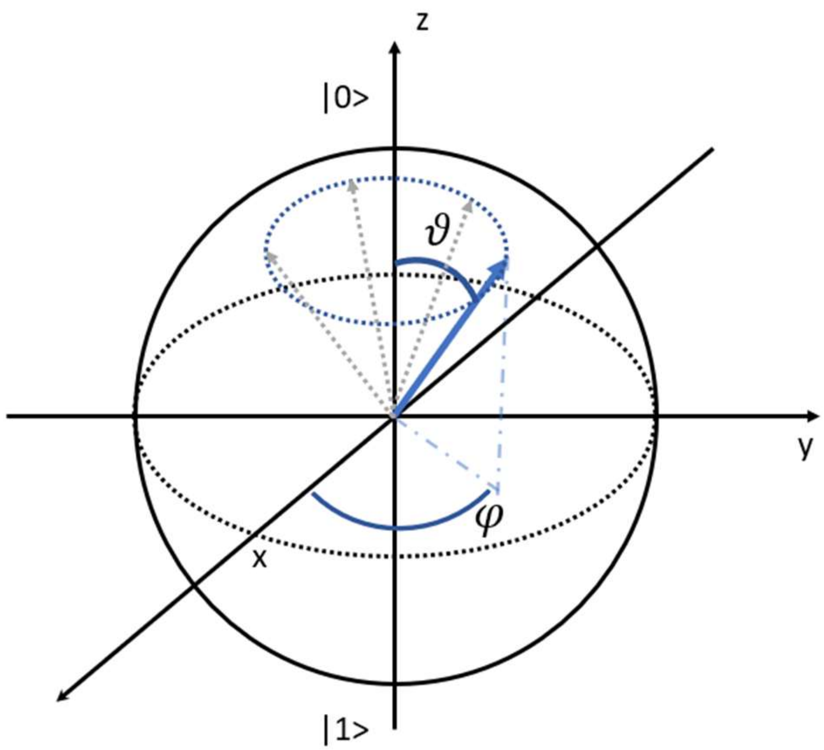
Eigenvektor

$$|x\rangle = \cos\left({}^{\vartheta}/_{2}\right)\begin{pmatrix}1\\0\end{pmatrix} + e^{i\varphi}\sin\left({}^{\vartheta}/_{2}\right)\begin{pmatrix}0\\1\end{pmatrix}$$



$$U\,|x> = U\begin{pmatrix}\cos\left({}^{\vartheta}/_{2}\right)\\ e^{i\varphi}\sin\left({}^{\vartheta}/_{2}\right)\end{pmatrix} = e^{i\alpha}\begin{pmatrix}\cos\left({}^{\vartheta}/_{2}\right)\\ e^{i\varphi}\sin\left({}^{\vartheta}/_{2}\right)\end{pmatrix} = \begin{pmatrix}\cos\left({}^{\vartheta}/_{2}\right)\\ e^{i\sigma}\sin\left({}^{\vartheta}/_{2}\right)\end{pmatrix}$$

Eigenvektor



$$U\,|x> = \begin{pmatrix} \cos\left(\vartheta/_2\right) \\ e^{i\sigma}\sin\left(\vartheta/_2\right) \end{pmatrix}$$

QPE

$U\,|x> = \lambda\;|x>$

$\lambda = e^{i\,\alpha}$

$$U^j\,|x> = \lambda^j\;\Big|x> = e^{i\,j\alpha}\,|x>$$

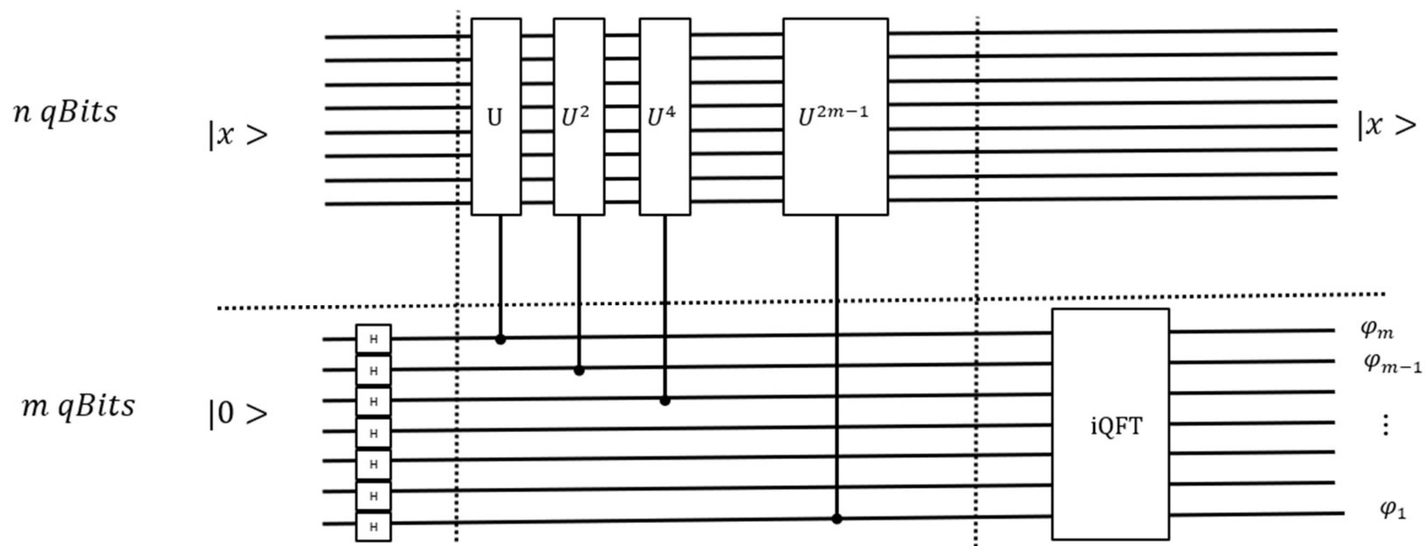
QPE

$U\,|x> = \lambda\,|x>$ $\lambda = e^{2\pi\ \varphi}$ $0 \leq \varphi < 1$

$$\varphi = \sum_{k=1}^{m} \frac{\varphi_k}{2^k}$$ $\varphi_k \in \{0,1\}$

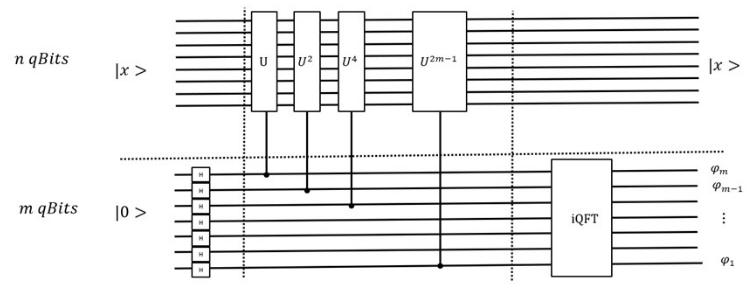
$$U^k\,|x> = \lambda^k\,|x> = e^{2\pi i\, k\varphi}\,|x>$$

Quantum Circuit



$$U^k\ |x> = \lambda^k\ \ |x> =\ e^{2\pi i\ k\varphi}\ |x>$$

QPE



$$|++ + \cdots +>\ |x>$$

$$\underbrace{\frac{1}{\sqrt{2^m}}(|0> + e^{2\pi i\ 0.\varphi_1 \ldots \varphi_m}\ \ |1>) \otimes (|0> + e^{2\pi i\ 0.\varphi_2 \ldots \varphi_m}\ \ |1>) \ldots \otimes (|0> + e^{2\pi i\ \ 0.\varphi_m}\ \ |1>)}\otimes |x>$$

$$QFT\ |\varphi_1 \ldots \varphi_m>$$

$$|\varphi_1 \ldots \varphi_m>\ |x>$$

$$\varphi = \frac{\varphi_1}{2} + \frac{\varphi_2}{4} + \cdots + \frac{\varphi_m}{2^m}$$

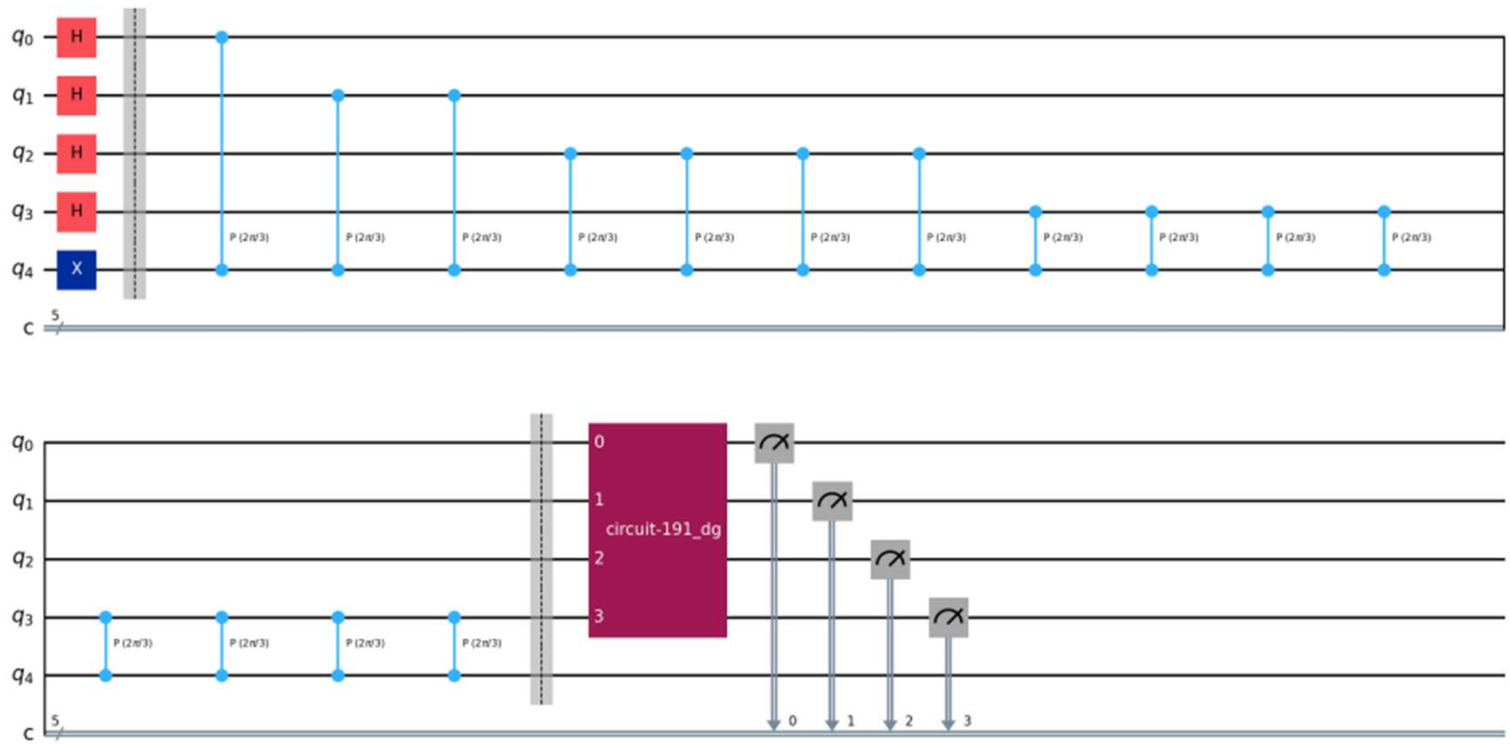
QPE

$$\theta = \frac{2}{3}\pi$$

$$P(\theta) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$$

$$P\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$P\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ e^{i\theta} \end{pmatrix} = e^{i\theta}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

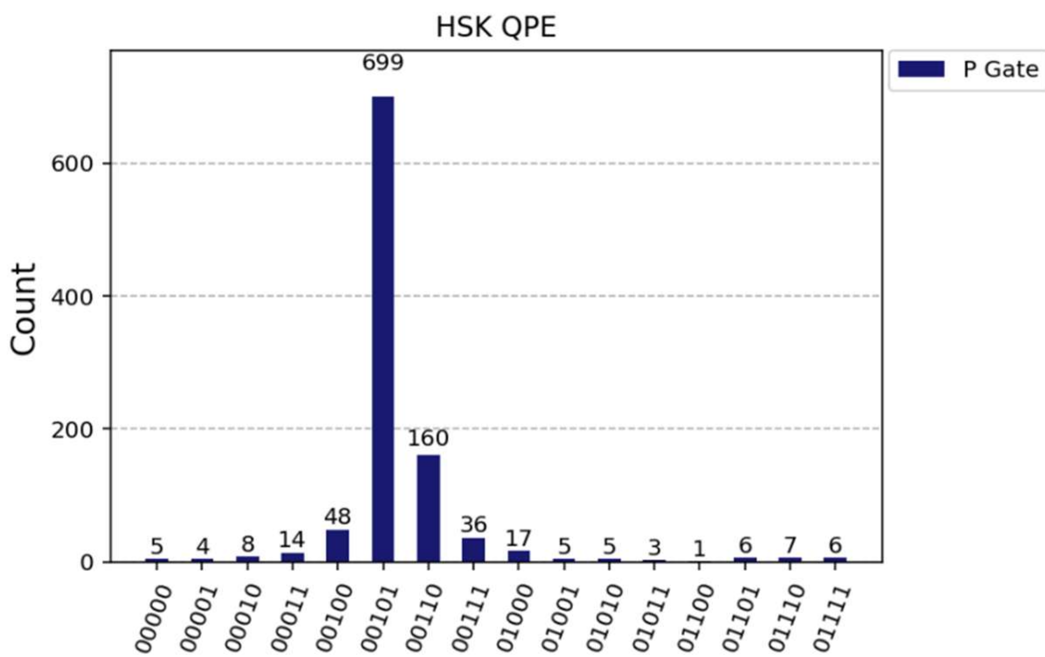


QPE

$$P(\theta) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$$

$$\theta = \frac{2}{3}\pi = 2.09$$

$$P\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ e^{i\theta} \end{pmatrix} = e^{i\theta}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



$$\varphi = \frac{0}{2} + \frac{1}{4} + \frac{0}{8} + \frac{1}{16} = \frac{5}{16}$$

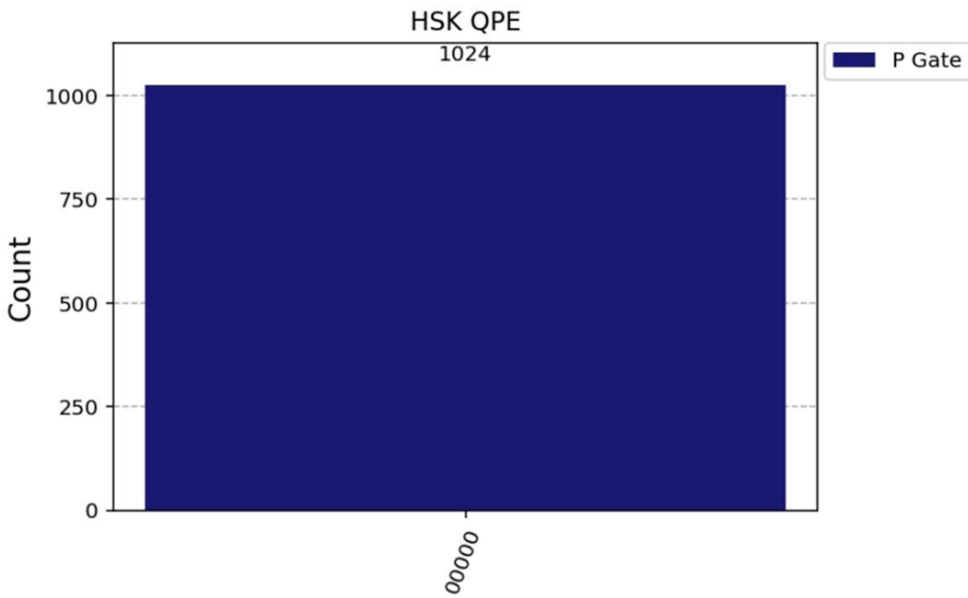
$$\theta_{est.} = 1{,}963$$

QPE

$$P(\theta) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$$

$$\theta = \frac{2}{3}\pi = 2.09$$

$$P\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$



$$\varphi = \frac{0}{2} + \frac{0}{4} + \frac{0}{8} + \frac{0}{16} = 0$$

$$\theta_{est.} = 0$$