EQC – Lösungen 9

Aufgabe 1:

Gegeben seien die Primzahlen x=7 und y=11, sowie die ganze Zahl a=8

ggT(77,8)

PFZ 77: 7 * 11

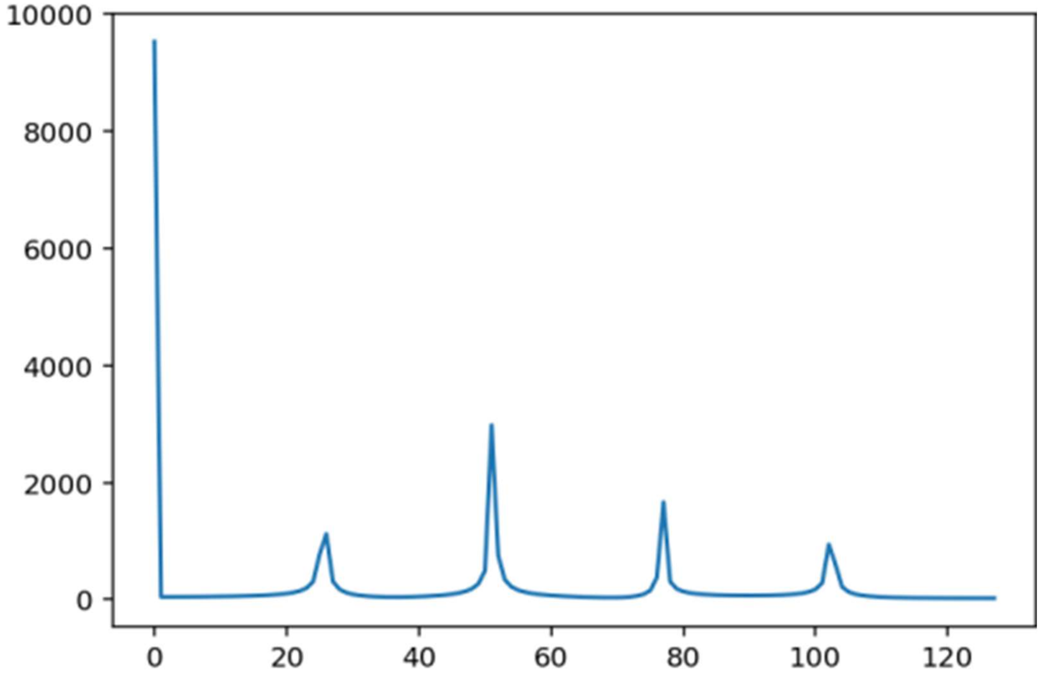
PFZ 8: 2^3

- ➔ ggT(77,8) = 1

Zahlenreihe:

k: 0 Row Value: 1
k: 1 Row Value: 8
k: 2 Row Value: 64
k: 3 Row Value: 50
k: 4 Row Value: 15
k: 5 Row Value: 43
k: 6 Row Value: 36
k: 7 Row Value: 57
k: 8 Row Value: 71
k: 9 Row Value: 29
k: 10 Row Value: 1
k: 11 Row Value: 8
k: 12 Row Value: 64
k: 13 Row Value: 50
k: 14 Row Value: 15
k: 15 Row Value: 43
k: 16 Row Value: 36
k: 17 Row Value: 57
k: 18 Row Value: 71
k: 19 Row Value: 29
k: 20 Row Value: 1
k: 21 Row Value: 8
k: 22 Row Value: 64
k: 23 Row Value: 50

FFT n=8, 256 Punkte



i: 26 Peak: 7.019514692115206

i: 51 Peak: 7.999529969175394

i: 77 Peak: 7.416514940141585

i: 102 Peak: 6.845424179205048

256 : 26 = 9.84

256 : 51 = 5.01

256 : 77 = 3.32

256 : 102 = 2.5

Periodizität p=10

Aufgabe 2:

Bestimme das multiplikative Inverse modulo m (ab mod m = 1) für folgende Tafel:

a/m	6	7	8	9
3	x	5	3	x
4	x	2	8	4
5	5	3	5	2
6	x	6	x	x

Aufgabe 3:

$R_m = \begin{pmatrix} 1 & 0 \\ 0 & \omega_m \end{pmatrix}$ $\omega_m = e^{i \cdot 2\pi/m}$ m = 4

- ω_m^0 , ω_m^1 , ω_m^2 , ω_m^3

$\omega_4^0 = 1$

$\omega_4^1 = e^{i \cdot \pi/2} = i$

$\omega_4^2 = e^{i \cdot \pi} = -1$

$\omega_4^3 = e^{i \cdot 3\pi/2} = -i$

- $\mathrm{R} = R_m \otimes R_m$

$$R = R_4 \otimes R_4 = \begin{pmatrix} 1 & 0 \\ 0 & \omega_4 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & \omega_4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \omega_4 & 0 & 0 \\ 0 & 0 & \omega_4 & 0 \\ 0 & 0 & 0 & \omega_4^2 \end{pmatrix}$$

- $R_m \, |0>$

$R_4 \, |0> = \, |0>$

- $R_m \, |1>$

$R_4 \, |1> = \, \omega_4 \, |1>$

- R $|00>$, R $|01>$, R $|10>$, R $|11>$

$$R\begin{pmatrix}1\\0\\0\\0\end{pmatrix} = \begin{pmatrix}1 & 0 & 0 & 0\\0 & \omega_4 & 0 & 0\\0 & 0 & \omega_4 & 0\\0 & 0 & 0 & \omega_4^2\end{pmatrix}\begin{pmatrix}1\\0\\0\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$$

$$R\begin{pmatrix}0\\1\\0\\0\end{pmatrix} = \begin{pmatrix}1 & 0 & 0 & 0\\0 & \omega_4 & 0 & 0\\0 & 0 & \omega_4 & 0\\0 & 0 & 0 & \omega_4^2\end{pmatrix}\begin{pmatrix}0\\1\\0\\0\end{pmatrix} = \begin{pmatrix}0\\\omega_4\\0\\0\end{pmatrix} = \begin{pmatrix}0\\i\\0\\0\end{pmatrix}$$

$$R\begin{pmatrix}0\\0\\1\\0\end{pmatrix} = \begin{pmatrix}1 & 0 & 0 & 0\\0 & \omega_4 & 0 & 0\\0 & 0 & \omega_4 & 0\\0 & 0 & 0 & \omega_4^2\end{pmatrix}\begin{pmatrix}0\\0\\1\\0\end{pmatrix} = \begin{pmatrix}0\\0\\\omega_4\\0\end{pmatrix} = \begin{pmatrix}0\\0\\i\\0\end{pmatrix}$$

$$R\begin{pmatrix}0\\0\\0\\1\end{pmatrix} = \begin{pmatrix}1 & 0 & 0 & 0\\0 & \omega_4 & 0 & 0\\0 & 0 & \omega_4 & 0\\0 & 0 & 0 & \omega_4^2\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\\\omega_4^2\end{pmatrix} = \begin{pmatrix}0\\0\\0\\-1\end{pmatrix}$$

Aufgabe 4:

Für

$\omega_N = e^{i.2\pi}/_N$

und N = 4:

$$W = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & \omega_4^1 & \omega_4^2 & \omega_4^3 \\ 1 & \omega_4^2 & \omega_4^4 & \omega_4^6 \\ 1 & \omega_4^3 & \omega_4^6 & \omega_4^9 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix}$$

W unitär :

$$W^\dagger = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{pmatrix}$$

$$WW^\dagger = \frac{1}{4}\begin{pmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- W $|00>$, W $|01>$, W $|10>$, W $|11>$

$$W\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$W\begin{pmatrix}0\\1\\0\\0\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1 & 1 & 1 & 1\\1 & i & -1 & -i\\1 & -1 & 1 & -1\\1 & -i & -1 & i\end{pmatrix}\begin{pmatrix}0\\1\\0\\0\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1\\i\\-1\\-i\end{pmatrix}$$

$$W\begin{pmatrix}0\\0\\1\\0\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1 & 1 & 1 & 1\\1 & i & -1 & -i\\1 & -1 & 1 & -1\\1 & -i & -1 & i\end{pmatrix}\begin{pmatrix}0\\0\\1\\0\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1\\-1\\1\\-1\end{pmatrix}$$

$$W\begin{pmatrix}0\\0\\0\\1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1 & 1 & 1 & 1\\1 & i & -1 & -i\\1 & -1 & 1 & -1\\1 & -i & -1 & i\end{pmatrix}\begin{pmatrix}0\\0\\0\\1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1\\-i\\-1\\i\end{pmatrix}$$