EQC – Lösungen 8

Aufgabe 1:

$$|x>=\begin{pmatrix}\cos\alpha\\ \sin\alpha\end{pmatrix}$$

$$|y>=\begin{pmatrix}-\sin\alpha\\ \cos\alpha\end{pmatrix}$$

$$|\phi^- >=\frac{1}{\sqrt{2}}(|\mathrm{xx}\rangle-|\mathrm{yy}\rangle)=\frac{1}{\sqrt{2}}\left(\begin{pmatrix}cos\alpha^2\\ cos\alpha\ sin\alpha\\ sin\alpha\ cos\alpha\\ sin\alpha^2\end{pmatrix}-\begin{pmatrix}sin\alpha^2\\ -sin\alpha\ cos\alpha\\ -cos\alpha\ sin\alpha\\ cos\alpha^2\end{pmatrix}\right)$$

$$=\frac{1}{\sqrt{2}}\left(\begin{pmatrix}1-\ 2\ sin\alpha^2\\ sin2\alpha\\ sin2\alpha\\ 2\ sin\alpha^2-1\end{pmatrix}\right)$$

$\boldsymbol{\alpha=0}$ **:**

$$=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\-1\end{pmatrix}=\frac{1}{\sqrt{2}}\ (|00>\ -|11>\ )=\ |\phi^->$$

$\boldsymbol{\alpha={}^{\pi}/_{2}}$ **:**

$$=\frac{1}{\sqrt{2}}\begin{pmatrix}-1\\0\\0\\1\end{pmatrix}=\frac{1}{\sqrt{2}}\ (-|00>\ +|11>\ )=\ -|\phi^->$$

$$|\psi^+ >=\frac{1}{\sqrt{2}}(|\mathrm{xy}\rangle+|\mathrm{yx}\rangle)=\frac{1}{\sqrt{2}}\left(\begin{pmatrix}-\sin\alpha\cos\alpha\\ cos\alpha^2\\ -\sin\alpha^2\\ sin\alpha\ cos\alpha\end{pmatrix}+\begin{pmatrix}-cos\alpha\ sin\alpha\\ -\sin\alpha^2\\ cos\alpha^2\\ sin\alpha\ cos\alpha\end{pmatrix}\right)$$

$$= \frac{1}{\sqrt{2}} \left(\begin{pmatrix} -sin2\alpha \\ 1 - \ 2\ sin\alpha^2 \\ 1 - \ 2\ sin\alpha^2 \\ sin2\alpha \end{pmatrix} \right)$$

$\boldsymbol{\alpha = 0}$ **:**

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \ (|01 > \ + |10 > \ ) = \ |\psi^+ >$$

$\boldsymbol{\alpha = {}^{\pi}/_{2}}$ **:**

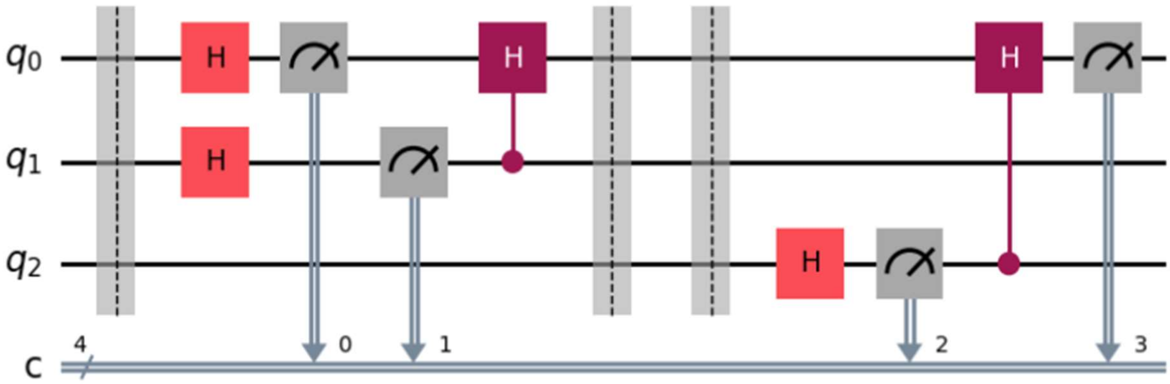
$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ -1 \\ -1 \\ 0 \end{pmatrix} = \frac{-1}{\sqrt{2}} \ (|01 > \ + |10 > \ ) = \ - |\psi^+ >$$

$$|\psi^- >= \frac{1}{\sqrt{2}} (|\mathrm{xy}\rangle - |\mathrm{yx}\rangle) = \frac{1}{\sqrt{2}} \left(\begin{pmatrix} -\sin\alpha\cos\alpha \\ cos\alpha^2 \\ -\sin\alpha^2 \\ sin\alpha\ cos\alpha \end{pmatrix} - \begin{pmatrix} -cos\alpha\ sin\alpha \\ -\sin\alpha^2 \\ cos\alpha^2 \\ sin\alpha\ cos\alpha \end{pmatrix} \right)$$

$$= \frac{1}{\sqrt{2}} \left(\begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix} \right) = \frac{1}{\sqrt{2}} \ (|01 > \ - |10 > \ ) = \ |\psi^- >$$

Aufgabe 2:

Quantenschaltkreis:



Wahrscheinlichkeiten:

$|000 >$

$\frac{1}{2}\left(|000 > + |010 > + |100 > + |110 >\right)$

P(M0 = 0) $= \frac{2}{4}$

P(M0 = 1) $= \frac{2}{4}$

P(M1 = 0) $= \frac{2}{4}$

P(M1 = 1) $= \frac{2}{4}$

$$\frac{1}{2}\left(\Big|000 > + \frac{1}{\sqrt{2}}\left(\Big|010 > +|110 >\right) + \Big|100 > + \frac{1}{\sqrt{2}}\left(|010 > - \Big|110 >\right)\right)$$

$$\frac{1}{2}\left(\Big|000 > + \frac{2}{\sqrt{2}}\Big|010 > +|100 >\right)$$

$$\frac{1}{2}\left(\frac{1}{\sqrt{2}}\left(|000 > +|001 >\right) + \left(|010 > +|011 >\right) + \frac{1}{\sqrt{2}}\left(|100 > + |101 >\right)\right)$$

P(M2 = 0) $= \frac{2}{8} + \frac{1}{4} = \frac{4}{8}$

P(M2 = 1) $= \frac{2}{8} + \frac{1}{4} = \frac{4}{8}$

$$= \frac{1}{\sqrt{8}}\Big|000> + \frac{1}{\sqrt{8}}\Big|001> + \frac{1}{2}\Big|010> + \frac{1}{2}\Big|011> + \frac{1}{\sqrt{8}}\Big|100> + \frac{1}{\sqrt{8}}\Big|101>$$

$$= \frac{1}{\sqrt{8}}|000\rangle + \frac{1}{4}(|001\rangle + |101\rangle) + \frac{1}{2}\Big|010\rangle + \frac{1}{\sqrt{8}}(|011\rangle + |111\rangle) + \frac{1}{\sqrt{8}}\Big|100\rangle + \frac{1}{4}(|001\rangle - |101\rangle)$$

$$= \frac{1}{\sqrt{8}}|000\rangle + \frac{1}{2}|001\rangle + \frac{1}{2}\Big|010\rangle + \frac{1}{\sqrt{8}}\Big|011\rangle + \frac{1}{\sqrt{8}}|111\rangle + \frac{1}{\sqrt{8}}|100\rangle$$

P(M3 = 0) $\frac{1}{8} + \frac{1}{4} + \frac{1}{4} + \frac{1}{8} = \frac{6}{8}$

P(M3 = 1) $\frac{1}{8} + \frac{1}{8} = \frac{2}{8}$

- M0 = M3

P1 = P(M0 = 0) x P(M3 = 0) + P(M0 = 1) x P(M3 = 1) = $\frac{1}{2} \cdot \frac{6}{8} + \frac{1}{2} \cdot \frac{2}{8} = \frac{4}{8}$

- M1 = M2

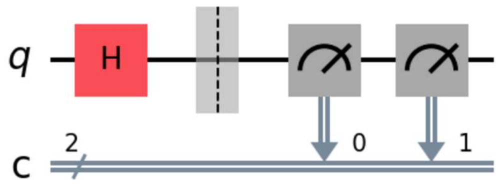
P2 = P(M1 = 0) x P(M2 = 0) + P(M1 = 1) x P(M1 = 1) = $\frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$

- M0 = M3 und M1 = M2

P1 x P2 = $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

Aufgabe 3:

- A



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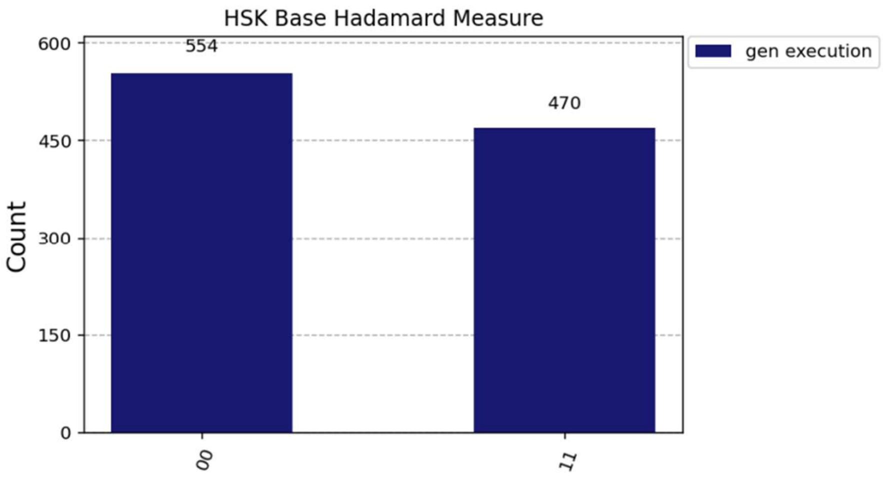

HSK Base Hadamard Measure
gen execution
Count
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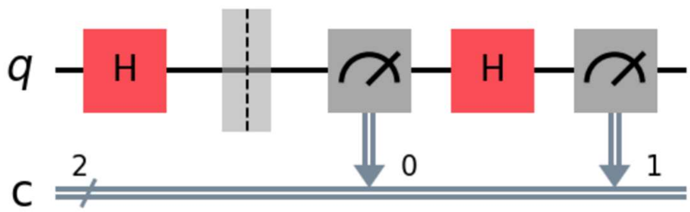
450
300
150
0
554
470
00
11

- B



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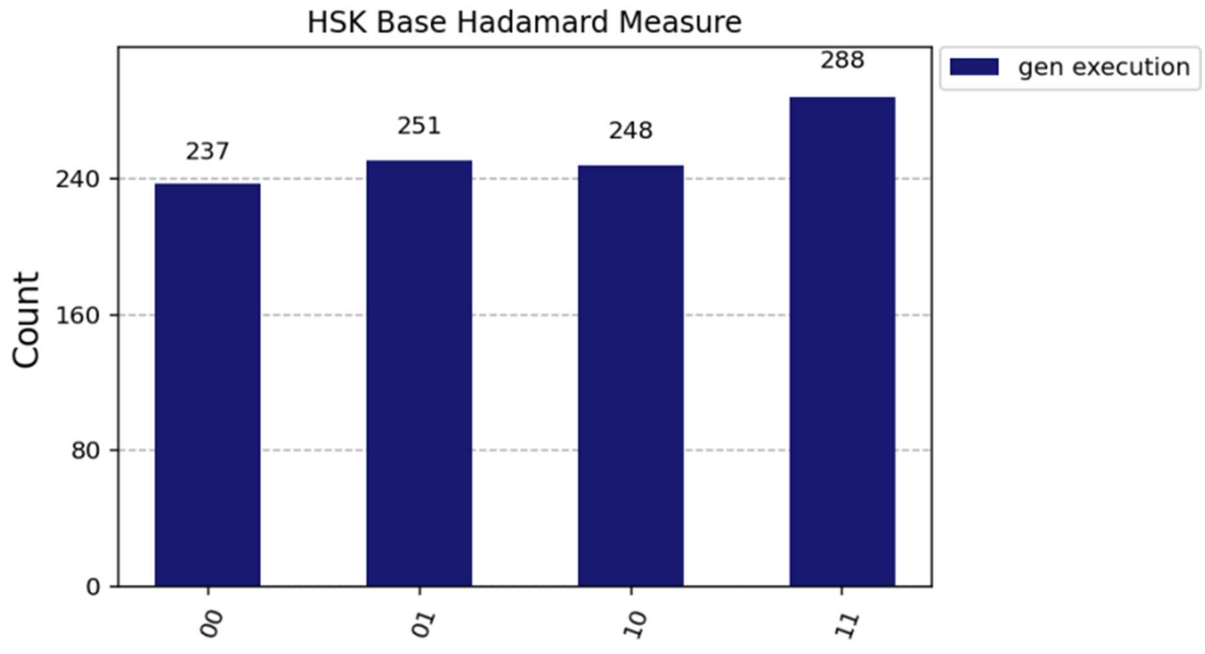
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01
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