Einführung

Quantum Computing

HSK 20.11.2024

- **Grover Algorithmus**
  - **Oracle**
  - **Diffusor**
  - **Messung**
  - **Outer Product**

Oracle

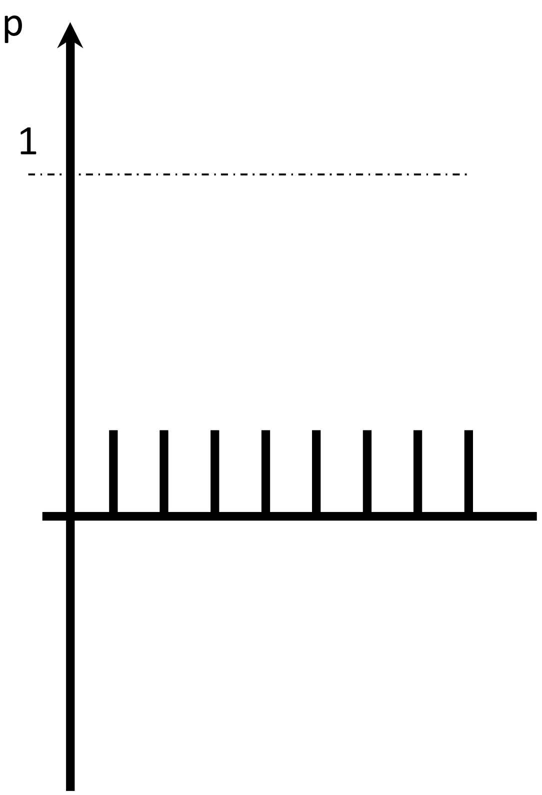
$$f:\{0,1\}^n \to \{0,1\}$$

$$\forall x \in \{0,1\}^n: \quad \exists!\, x_0 \quad f(x_0) = 1$$

$$U_f(|x>|y>) = |\,\mathrm{x}>|\mathrm{y} \oplus f(x)\ >$$

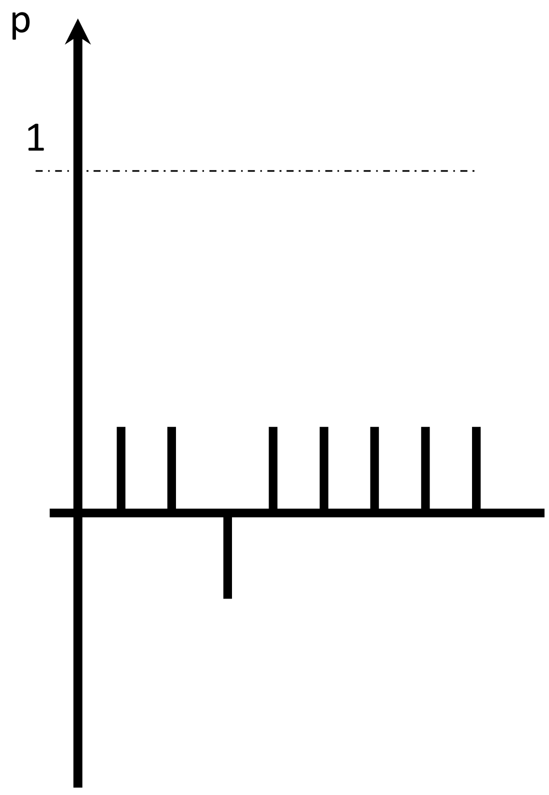
$$U_f(|x>\otimes|->) = (-1)^{f(x)} \cdot (|x>|->)$$

$$U_f\ \left|x>\right. = (-1)^{f(x)} \left|x>\right.$$

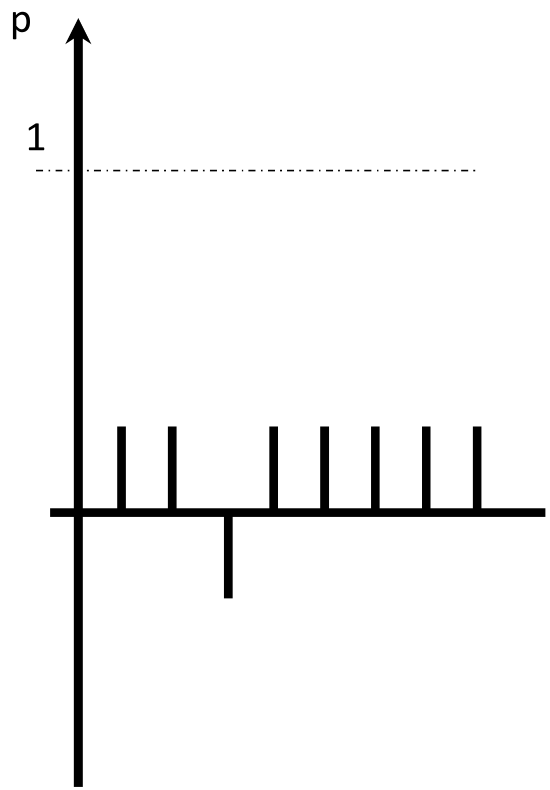


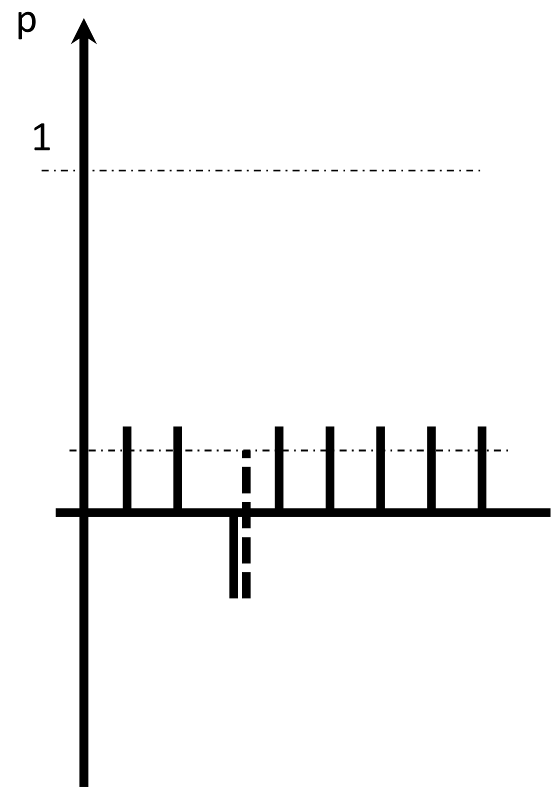
Diffusor

$$U_f \left| x > \right. = (-1)^{f(x)} \left| x > \right.$$

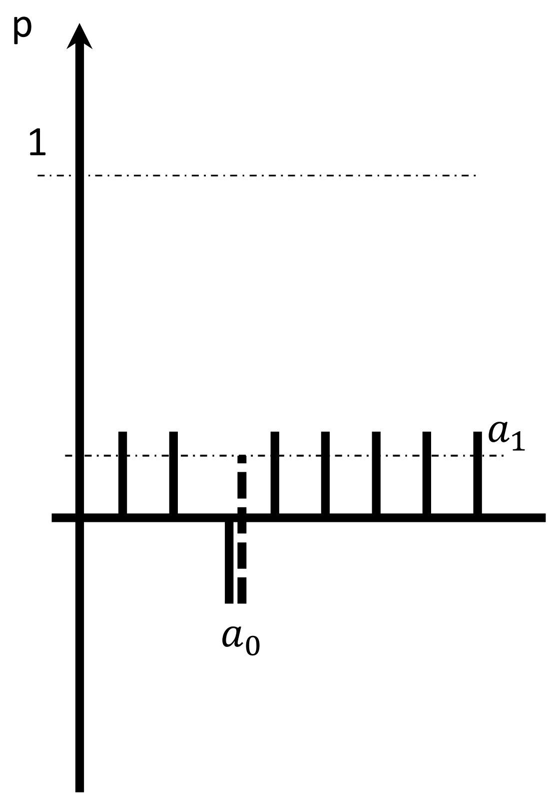


Diffusor



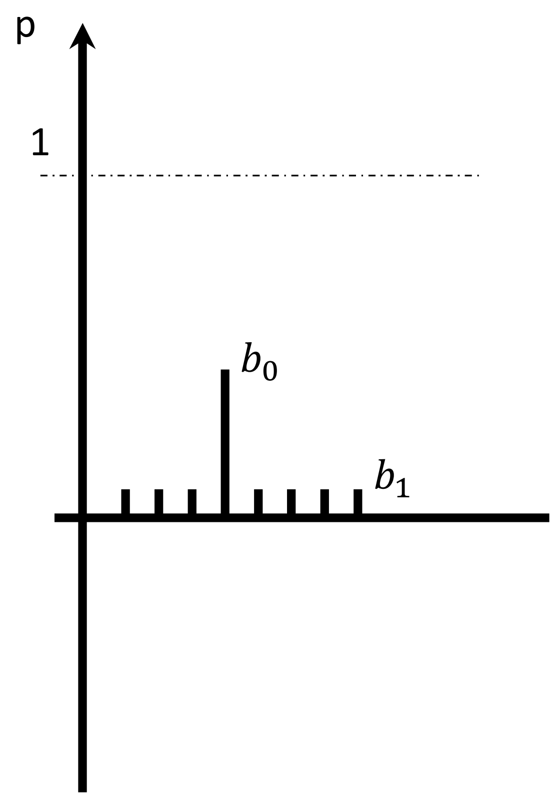


Diffusor



$$m = \frac{1}{N}\left((N-1)\cdot a_1 - a_0\right)$$

$$b = 2m - a$$



Grover

$$N = 2^n$$

$$|y> = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} | x_i >$$

$$|y> = \frac{1}{\sqrt{N}} \left(\sum_{i \neq k}^{N} | x_i > +| x_k > \right)$$

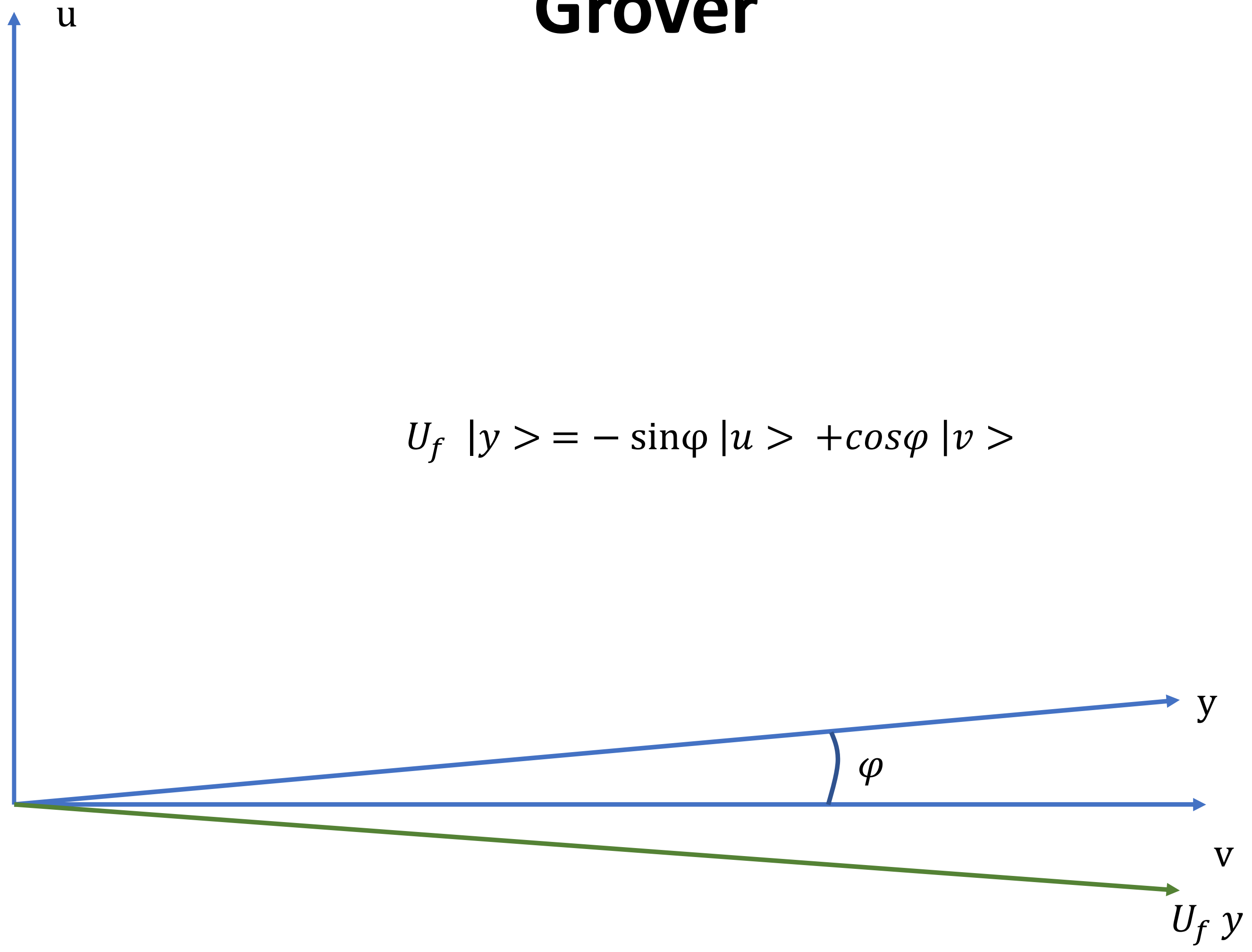
$$\frac{1}{\sqrt{N}} = \sqrt{\frac{N-1}{N}} \frac{1}{\sqrt{N-1}}$$

$$|y> = \frac{1}{\sqrt{N}} (| x_k >) + \sqrt{\frac{N-1}{N}} | x_{i \neq k} >$$

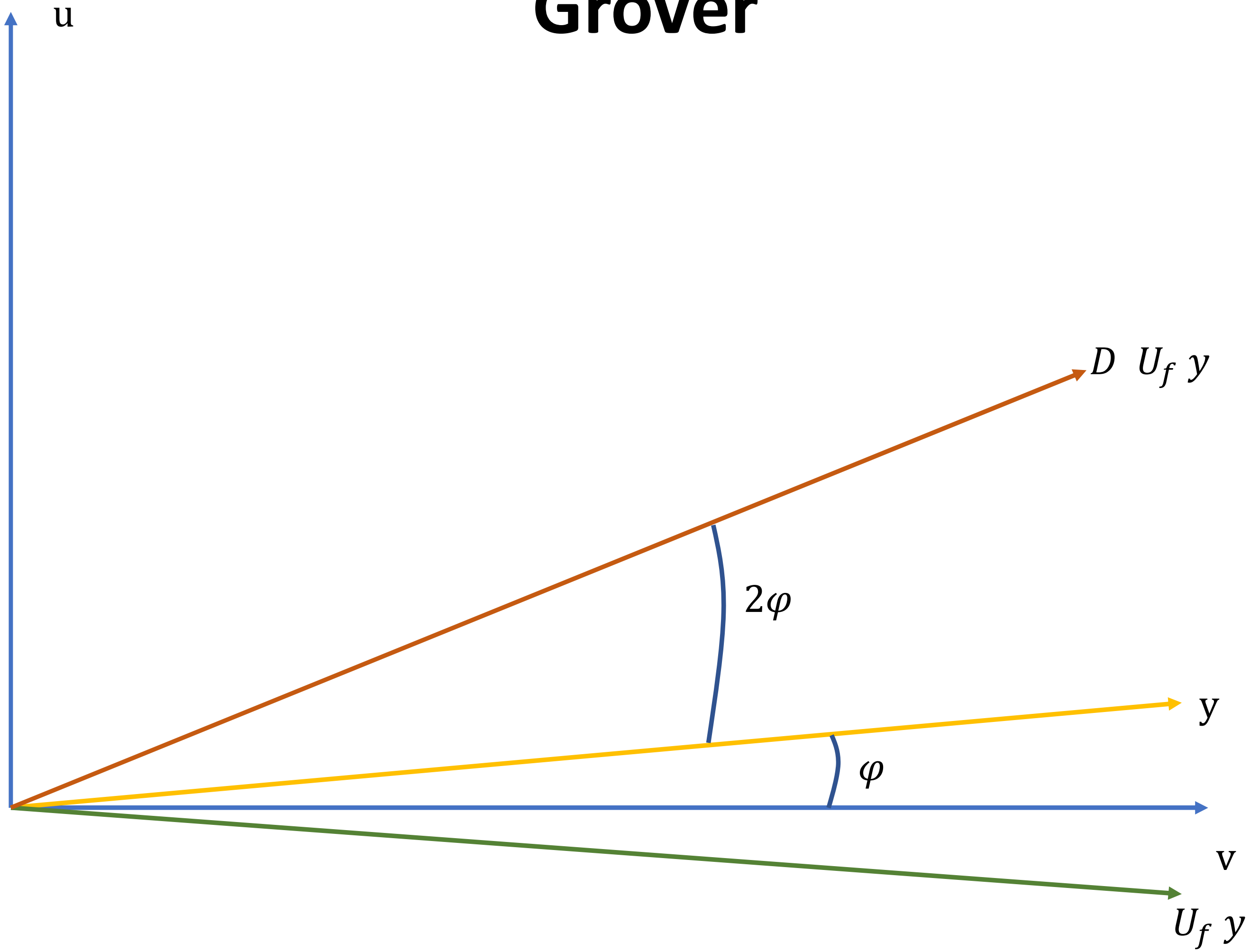
$$|y> = \sin\varphi \, |u> + cos\varphi \, |v>$$

Grover

$$U_f \ |y> = -\sin\varphi\ |u> \ + cos\varphi\ |v>$$

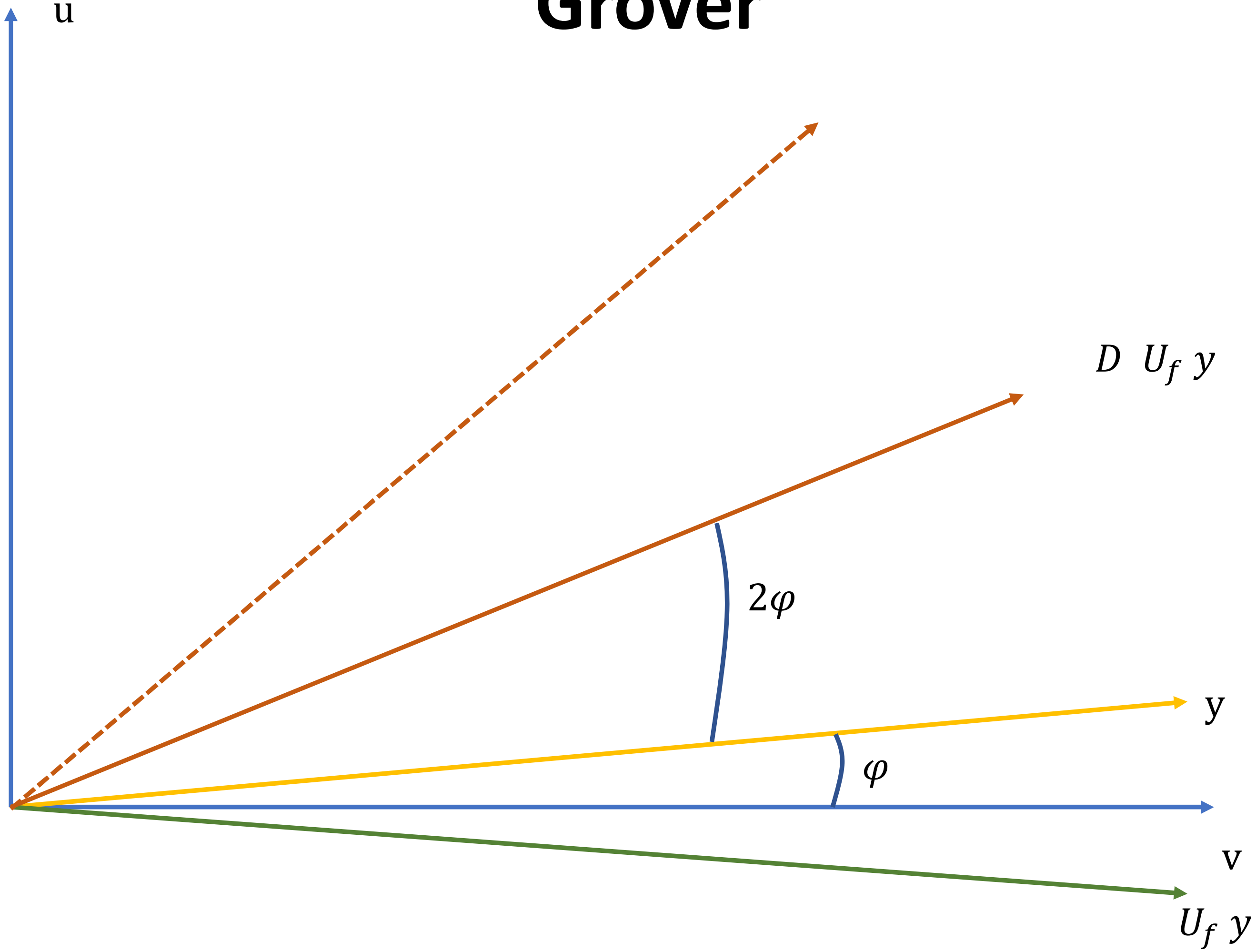


Grover

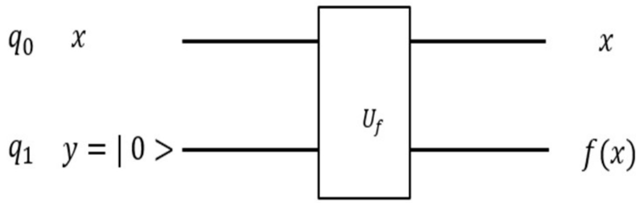


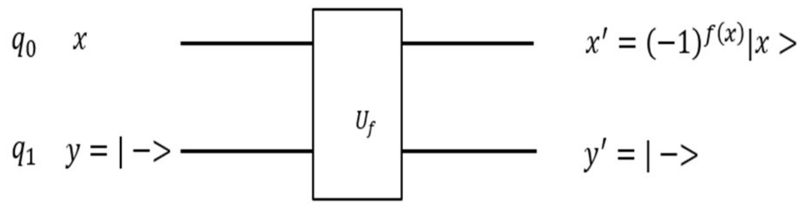
Grover

$$m = \frac{\pi}{4} \cdot \sqrt{N}$$



Deutsch Algorithmus - Oracle





$$N = 2^n$$

Diffusor

$$\sum_{i=1}^{N} a_i|i> \; \rightarrow \; \sum_{i=1}^{N}(2m - a_i)|i>$$

$$D_N = -H_n \cdot R_N \cdot H_n$$

$$R_N = \begin{matrix} -1 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & 1 \end{matrix}$$

$$f: \{0,1\}^n \rightarrow \{0,1\}$$

$$U_f(|x> \otimes |->) = (-1)^{f(x)} \cdot (|x> \otimes |->)$$

$R_N: |\mathrm{x}> \; |y> \rightarrow: |\mathrm{x}> \; |y \oplus f(x)>$ mit $\quad f(x_n \ldots \ldots . x_1) = \begin{cases} 1 \; f\text{ü}r \; x_n = \cdots = x_0 = 0 \\ 0 \; sonst \end{cases}$

Konjunktive Normalform

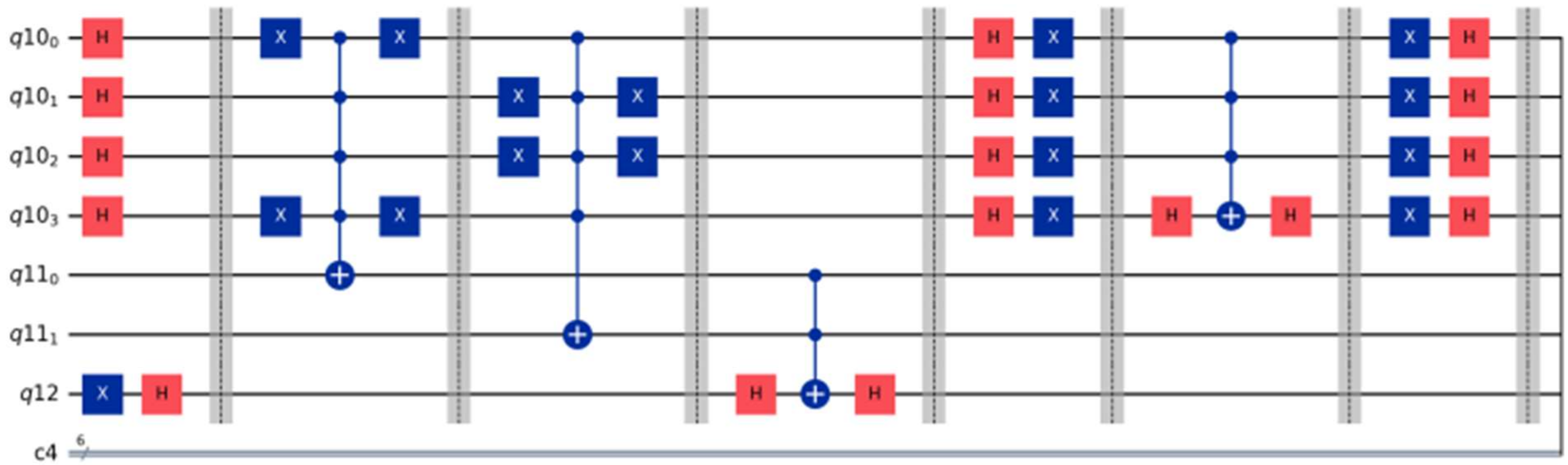
(A·~B)+(~A·B))·((C·~D)+(~C·D))·((A·~C)+(~A·C))·((B·~D)+(~B·D))

->

(A·D·~B·~C)+(B·C·~A·~D)

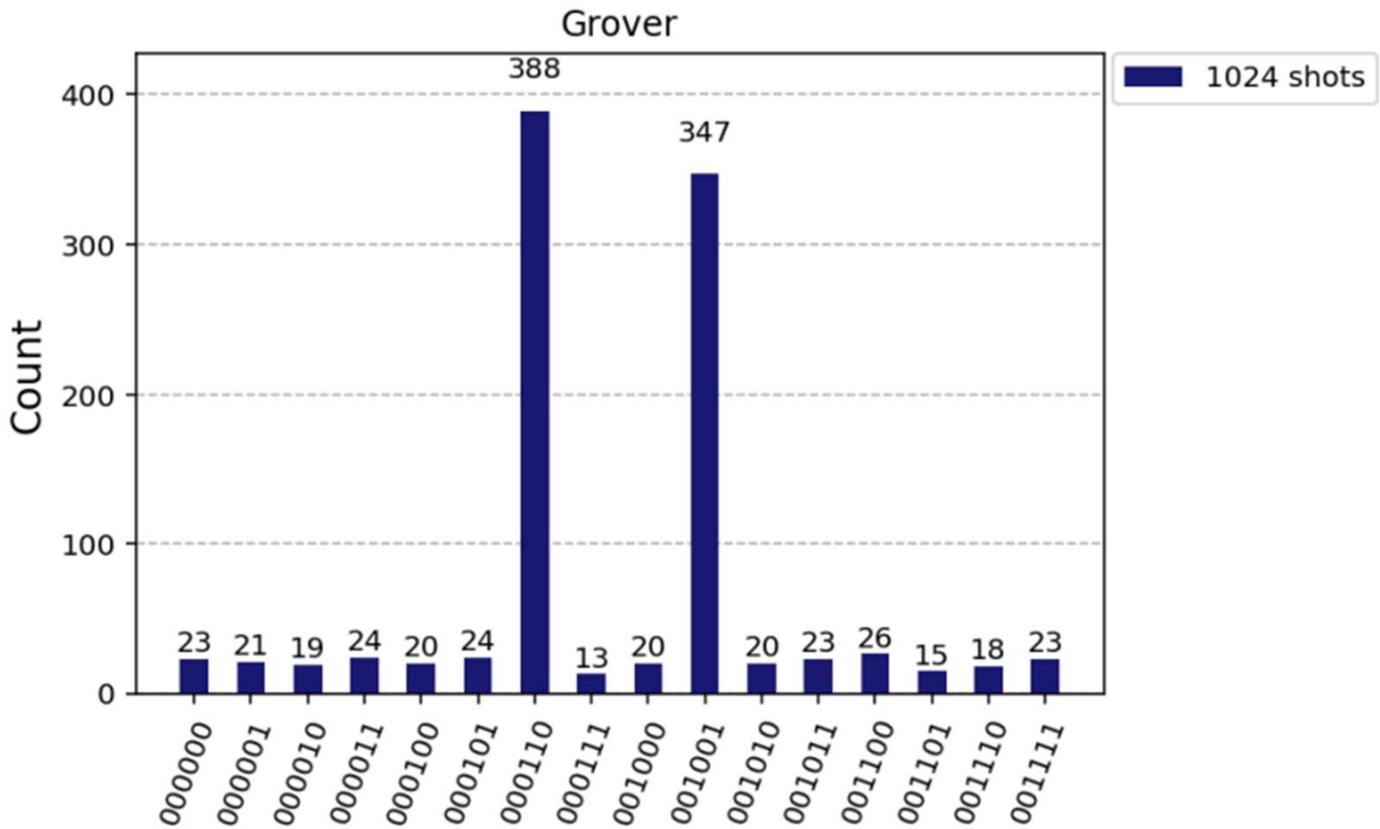
Konjunktive Normalform

(A ^ D ^ ~B ^ ~C) v (B ^ C ^ ~ A ^ ~D)



Konjunktive Normalform

(A ^ D ^ ~B ^ ~C) v (B ^ C ^ ~ A ^ ~D)



Outer Product

$$|x\rangle \cdot \langle y\,| = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \cdot (y_1^*, \dots., y_n^*) = \begin{pmatrix} x_1 \cdot y_1^* & \cdots & x_1 \cdot y_n^* \\ \vdots & \ddots & \vdots \\ x_n \cdot y_1^* & \cdots & x_n \cdot y_n^* \end{pmatrix}$$

$$I = \sum_{i=1}^{N} |i\rangle \cdot \langle i| = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 \end{pmatrix}$$

$\{\,|\,x >, |y >\}:$

$|\,x > < y\,|\,|\,y > = |x >$

$$N = 2^n$$

Diffusor

$$\sum_{i=1}^{N} a_i|i> \rightarrow \sum_{i=1}^{N}(2m - a_i)|i>$$

$$|s> = H\,|\,0>$$

$$2\,|\,s><s\,|\,-I = 2\,H\,|\,0><0\,|\,H - I$$

$$N = 2^n$$

Diffusor

$$\sum_{i=1}^{N} a_i|i> \rightarrow \sum_{i=1}^{N}(2m - a_i)|i>$$

$$|s> = H\,|\,0>$$

$$2\,|\,s><s\,|\, - I = 2\,(\,H\,|\,0><0\,|\,H\,)\, - \,I$$

$$I = HH$$

$$2\,(\,H\,|\,0><0\,|\,H\,) - HH = 2\,H\,(\,(\,|\,0><0\,|\,H\,)\, - H\,) = H\,(\,(\,2\,|0><0\,|\,)\, - I\,)H$$

$$R_N = (\,(\,2\,|0><0\,|\,)\, - I\,)$$

$$2\,|\,s><s\,|\, - I = HR_NH$$

$N = 2^n$

Diffusor

$2\,|\,s><s\,|\,-I = HR_NH$ $\quad$ $R_N = \left(\left(2\,|0><0\,| \right) - I \right)$ $\quad$ $|s>=H\,|\,0>$

$$R_N\,|\,0>=2\,|0><0\,|\,0> \;\; -|0>=|\,0>$$

$$R_N\,|\,a>=2\,|0><0\,|\,a> \;\; -|a>=-\,|\,a>$$

$$-R_N\colon \quad |\,0> \to -|0> \qquad |\,a> \to |\,a>$$