Einführung

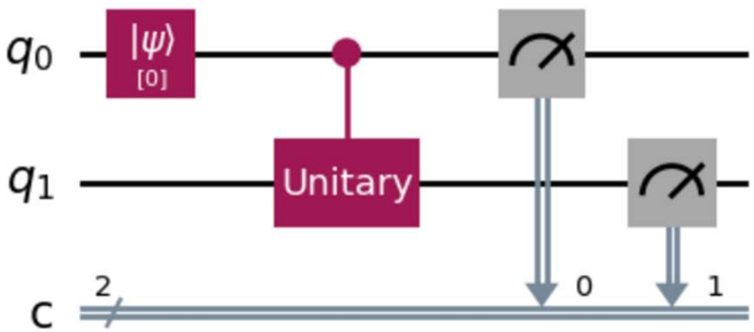
Quantum Computing

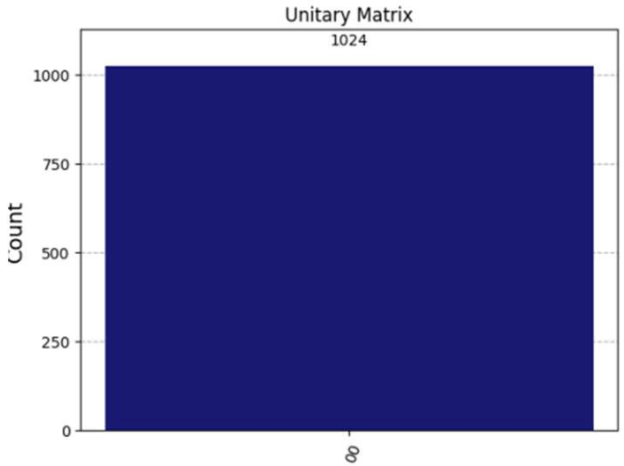
HSK 23.10.2024

- **Kontrollierte Gatter**
  - **CNOT**
- **No Cloning Theorem**
- **Deutsch Algorithmus**

Kontrollierte Unitäre Transformation

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$





Kontrollierte Unitäre Transformation

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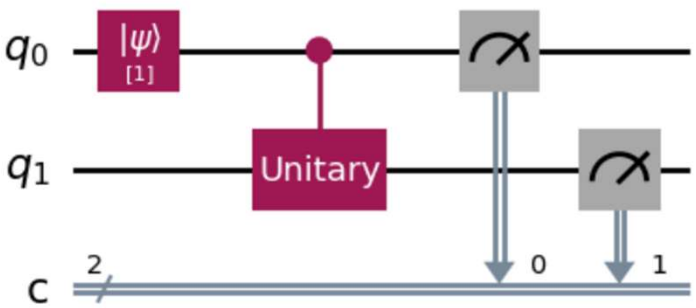

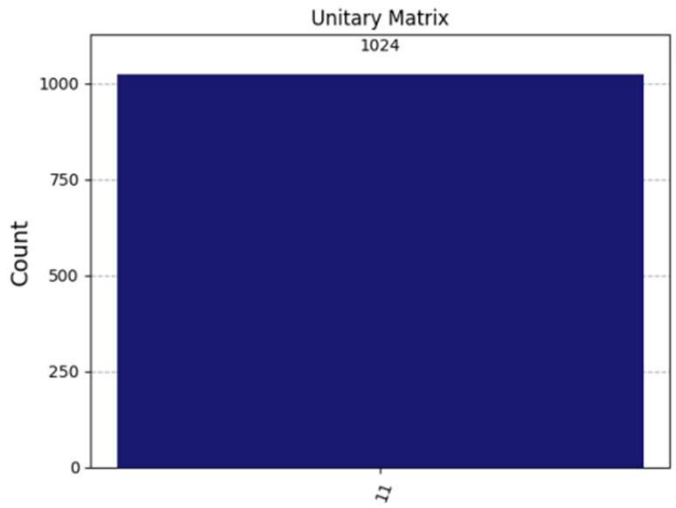

Kontrollierte Unitäre Transformation

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\text{CNOT} = \begin{pmatrix} I & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & U \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

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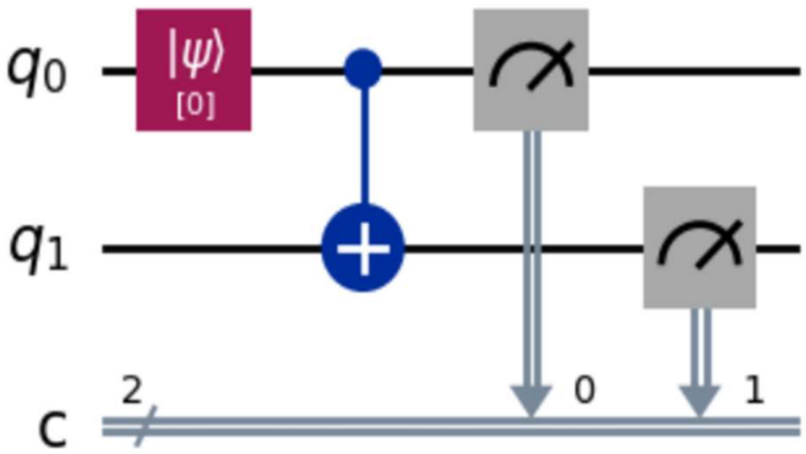
CNOT Gate



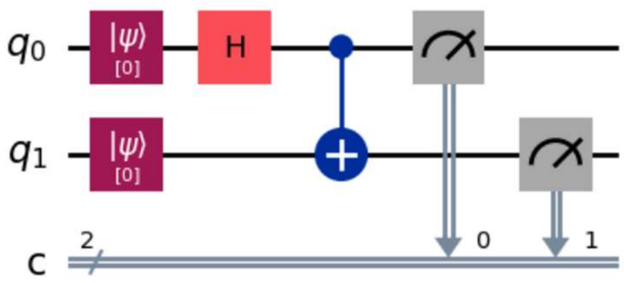
Bell States

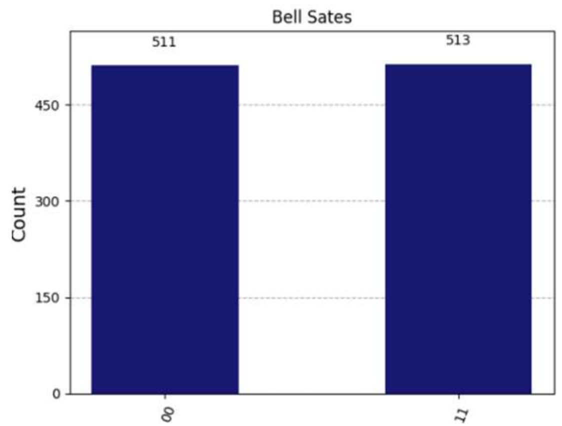
$$|\phi^{+}>=\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)$$

$$|\phi^{-}>=\frac{1}{\sqrt{2}}(|00\rangle-|11\rangle)$$

$$|\psi^{+}>=\frac{1}{\sqrt{2}}(|01\rangle+|10\rangle)$$

$$|\psi^{-}>=\frac{1}{\sqrt{2}}(|01\rangle-|10\rangle)$$





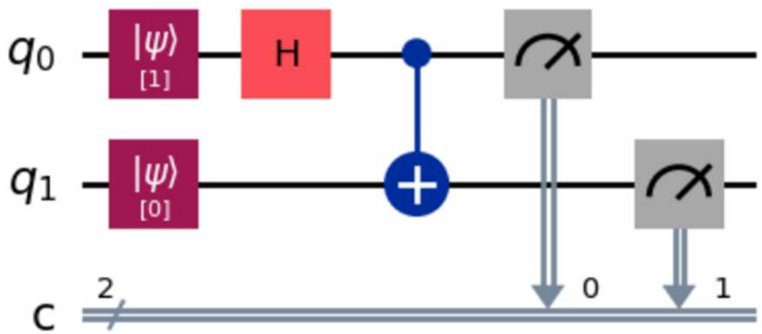
Bell States

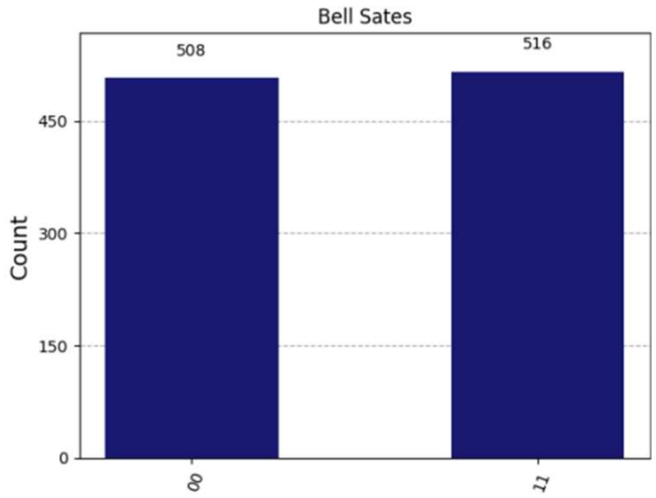
$$|\phi^+ >= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$|\phi^- >= \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

$$|\psi^+ >= \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

$$|\psi^- >= \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$





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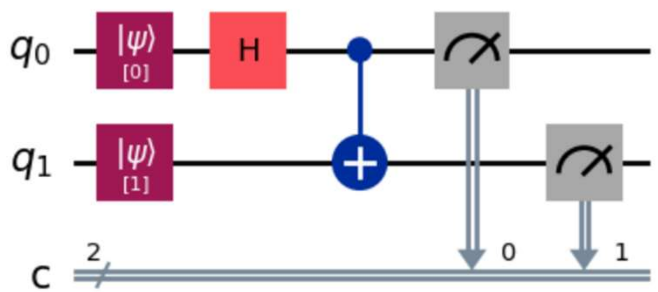

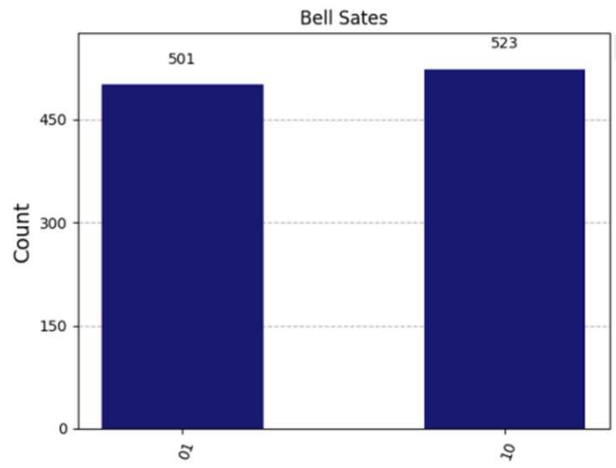

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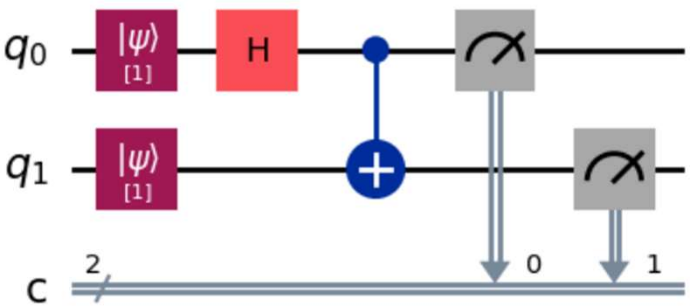
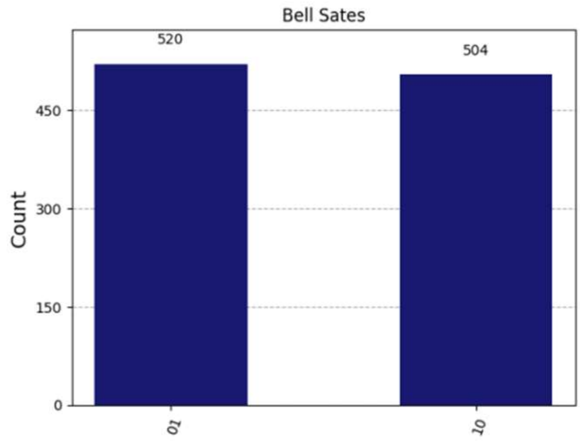
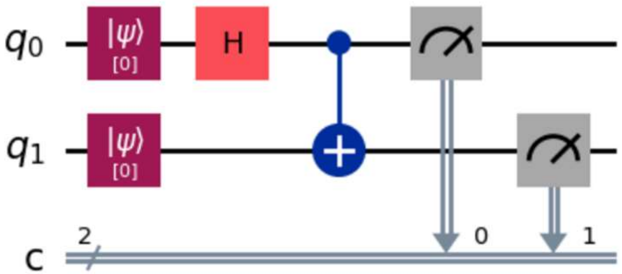
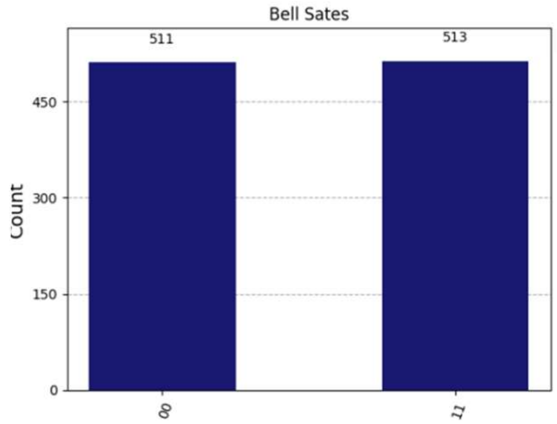




Bell States



$$|\phi^+ >= \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$



$$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

No Cloning Theorem

$\exists U, \phi$ so dass $\forall \psi$ gilt $U(|\psi> \otimes |\phi>) = (|\psi> \otimes |\psi>)$

$$|\phi> = |0>$$

$$U(|+> \otimes |0>) = (|+> \otimes |+>) = \frac{1}{2}(|00> + |01> + |10> + |11>)$$

No Cloning Theorem

$\exists U, \phi$ so dass $\forall \psi$ gilt $U(|\psi > \otimes|\phi >) = (|\psi > \otimes|\psi >)$

$$|\phi >= |0 >$$

$$U(|+> \otimes|0 >) = U\left(\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \right)\otimes|0 >\right) = \frac{1}{\sqrt{2}}U(|0 > \otimes |0 >) + \frac{1}{\sqrt{2}}U(|1 > \otimes|0 >)$$

$U(|0 > \otimes |0 >) = (|0 > \otimes |0 >) = |00 >$

$U(|1 > \otimes |0 >) = (|1 > \otimes |1 >) = |11 >$

$$U(|+> \otimes|0 >) = \frac{1}{\sqrt{2}}|00 > + \frac{1}{\sqrt{2}}|11 >$$

No Cloning Theorem

$\exists U, \phi$ so dass $\forall \psi$ gilt $U(|\psi> \otimes |\phi>) = (|\psi> \otimes |\psi>)$

$$|\phi> = |0>$$

$$U(|+> \otimes |0>) = (|+> \otimes |+>) = \frac{1}{2}(|00> + |01> + |10> + |11>)$$

$$U(|+> \otimes |0>) = \frac{1}{\sqrt{2}}|00> + \frac{1}{\sqrt{2}}|11>$$

Deutsch Algorithmus

$$f: \{0,1\} \rightarrow \{0,1\}$$

$$U_f: |\mathrm{xy}> \rightarrow |\mathrm{x}, y \oplus f(x)>$$

$\|\mathrm{xy}>$	$f(x) = 0$	$f(x) = 1$	$f(x) = x$	$f(x) = \neg x$
00	00	01	00	01
01	01	00	01	00
10	10	11	11	10
11	11	10	10	11

Deutsch Algorithmus

$$f: \{0,1\} \rightarrow \{0,1\}$$

$$U_f: |\mathrm{xy}> \rightarrow |\mathrm{x}, y \oplus f(x)>$$

$$f(x) = 0 \qquad U_f = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$f(x) = 1 \qquad U_f = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$f(x) = x \qquad U_f = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$f(x) = \neg x \qquad U_f = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Deutsch Algorithmus

$$f: \{0,1\} \rightarrow \{0,1\}$$

$$U_f: |\mathrm{xy}> \ \rightarrow |\mathrm{x}, y \oplus f(x)>$$

$|\mathrm{y}> = |0> \ :$

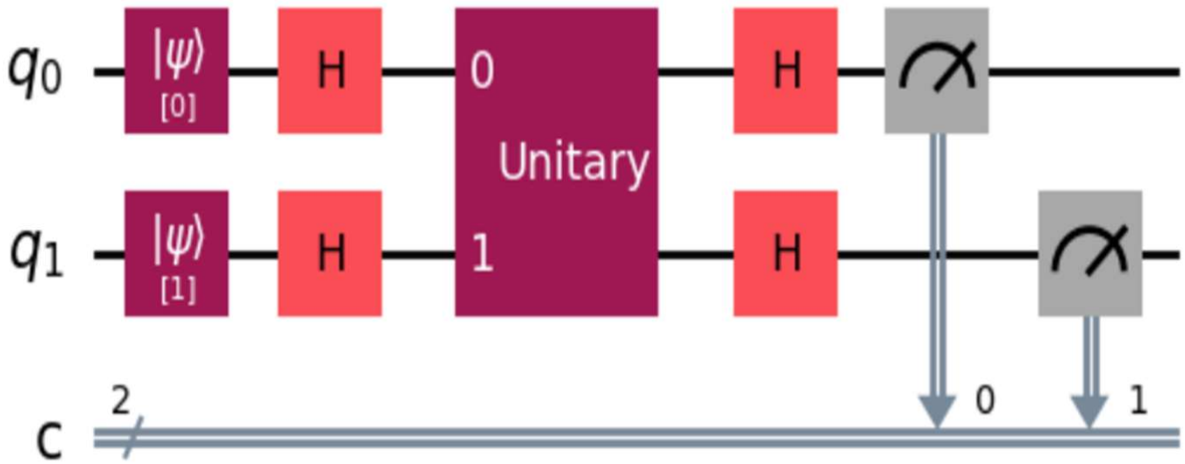
$\boldsymbol{x}$	$\boldsymbol{U_f}$	$\boldsymbol{x}$
$y = \|0>$		$f(x)$

$|\mathrm{y}> = |-> \ :$

$\boldsymbol{x}$	$\boldsymbol{U_f}$	$(-1)^{f(x)}\|x>$
$y = \|->$		$y = \|->$

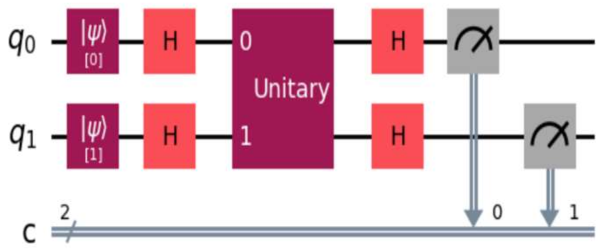
Deutsch Algorithmus

$$U_f : |\mathrm{xy} > \rightarrow |\mathrm{x}, y \oplus f(x) >$$



Deutsch Algorithmus

$U_f: |\mathrm{xy}> \; \rightarrow |\mathrm{x}, y \oplus f(x)>$



$$H_2|01>=|+>\cdot\;|->$$

$$|\mathrm{f}(x)> -|1 \oplus f(x)> = (-1)^{\mathrm{f}(x)} \cdot (|0> -|1>)$$

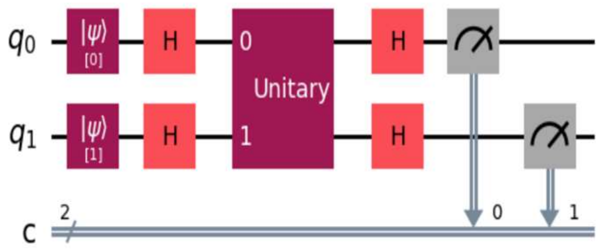
$$U_f(|+>\cdot\;|->) = \;|+>\cdot(-1)^{f(x)}|->$$

$$U_f(|+>\cdot\;|->) = \;(-1)^{f(x)}|+>\cdot|->$$

$$U_f(|+>\cdot\;|->) = \frac{1}{\sqrt{2}}\left((-1)^{f(0)}|0> +(-1)^{f(1)}|1>\right)\cdot|->$$

Deutsch Algorithmus

$U_f : |xy> \ \rightarrow |x, y \oplus f(x)>$



$U_f(|+>\cdot \ |->) = \frac{1}{\sqrt{2}}\left((-1)^{f(0)}|0> + (-1)^{f(1)}|1>\right) \cdot |->$

$f(0) = f(1)$:

$$U_f(|+>\cdot \ |->) = \frac{\pm 1}{\sqrt{2}} |+> \cdot |->$$

$f(0) \neq f(1)$:

Phase Kickback

$$U_f(|+>\cdot \ |->) = \frac{\pm 1}{\sqrt{2}} |-> \cdot |->$$