Einführung

Quantum Computing

HSK 16.10.2024

- **Quantenregister**
  - **Tensorprodukt**
  - **Messung eines QR**
  - **Verschränkung**

Tensorprodukt Vektoren

Vektorraum $\mathbb{C}^{\boldsymbol{n}}$

Vektoren $\vec{a}, \vec{b}$ ε $\mathbb{C}^{\boldsymbol{n}}$:

$$\vec{a} = \begin{pmatrix} a_1 \\ \cdots \\ a_n \end{pmatrix}, \vec{b} = \begin{pmatrix} b_1 \\ \cdots \\ b_n \end{pmatrix}$$

Tensorprodukt:

$$\vec{a} \otimes \vec{b} = \sum_{i=1}^{n} \sum_{j=1}^{n} a_i b_j \, |ij\rangle$$

2 qBits:

$$|x\rangle = \alpha_x|0\rangle + \beta_x|1\rangle \qquad |y\rangle = \alpha_y|0\rangle + \beta_y|1\rangle$$

$$\begin{pmatrix} \alpha_x \\ \beta_x \end{pmatrix} \otimes \begin{pmatrix} \alpha_y \\ \beta_y \end{pmatrix} = \begin{pmatrix} \alpha_x\alpha_y \\ \alpha_x\beta_y \\ \beta_x\alpha_y \\ \beta_x\beta_y \end{pmatrix}$$

Tensorprodukt Vektoren

Basiszustände: lexikographische Sortierung

Dirac:

$$|0> \otimes |0> = |00>$$

$$|0> \otimes |1> = |01>$$

$$|1> \otimes |0> = |10>$$

$$|1> \otimes |1> = |11>$$

Vektor:

$$\begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$$

$$\begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}$$

$$\begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\0\\1\\0\end{pmatrix}$$

$$\begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

Tensorprodukt Vektoren

Beliebiger Zustand:

$$|x\rangle = \alpha_x|0\rangle + \beta_x|1\rangle \qquad |y\rangle = \alpha_y|0\rangle + \beta_y|1\rangle$$

$$|x\rangle \otimes |y\rangle = (\alpha_x|0\rangle + \beta_x|1\rangle) \cdot \left(\alpha_y|0\rangle + \beta_y|1\rangle\right)$$

$$= \alpha_x\alpha_y\left|00 > +\alpha_x\beta_y\right|01 > +\beta_x\alpha_y|10 > +\beta_x\beta_y|11 >$$

Tensorprodukt Vektoren

n qBits:

$$|x_1\rangle = \alpha_1|0\rangle + \beta_1|1\rangle$$

$$\vdots$$

$$|x_n\rangle = \alpha_n|0\rangle + \beta_n|1\rangle$$

$$|x_1\rangle \otimes |x_2\rangle \ldots .. \otimes |x_n\rangle = \alpha_1\alpha_2 \cdots \alpha_n|00 \cdots 0 > +\alpha_1\alpha_2 \cdots \beta_n|00 \cdots 1 > \cdots \cdots + \beta_1 \cdots \beta_n|11 \cdots 11 >$$

2^n Zustände:

$$|0 \ldots \ldots .0 >, |0 \ldots \ldots 01 >, \ldots \ldots, |11 \ldots .10 >, |11 \ldots \ldots .. 11 >$$

Tensorprodukt Operatoren

Matrizen: A,B Ɛ $\mathbb{C}^{\boldsymbol{n\times n}}$

Tensorprodukt:

$$A\otimes B = \begin{pmatrix} a_{11}B & \dots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{n1}B & \dots & a_{nn}B \end{pmatrix}$$

Beispiel:

$$H_2 = H\otimes H = \frac{1}{\sqrt{2}}\begin{pmatrix} H & H \\ H & H \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

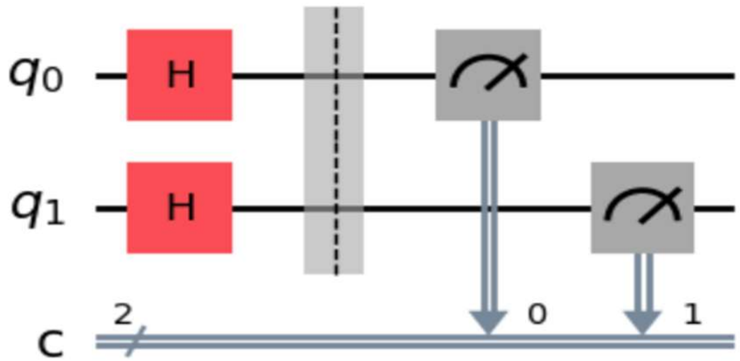
Tensorprodukt Rechenregeln

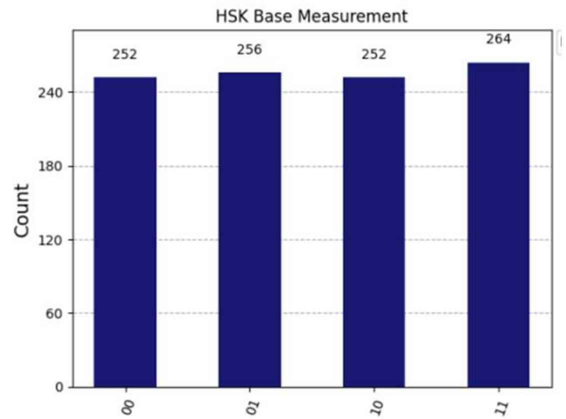
- $a \otimes b \neq b \otimes a$
- $a \otimes (b + c) = (a \otimes b) + (a \otimes c)$ und $(a + b) \otimes c = (a \otimes c) + (b \otimes c)$
- $(\lambda a) \otimes b = \lambda (a \otimes b) = a \otimes (\lambda b)$
- $a \otimes (b \otimes c) = (a \otimes b) \otimes c$

Tensorprodukt Operatoren

Q-Circuit:

$$H_2 = H \otimes H$$

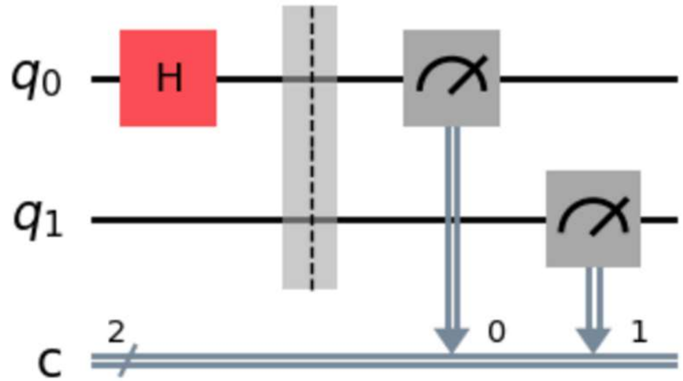


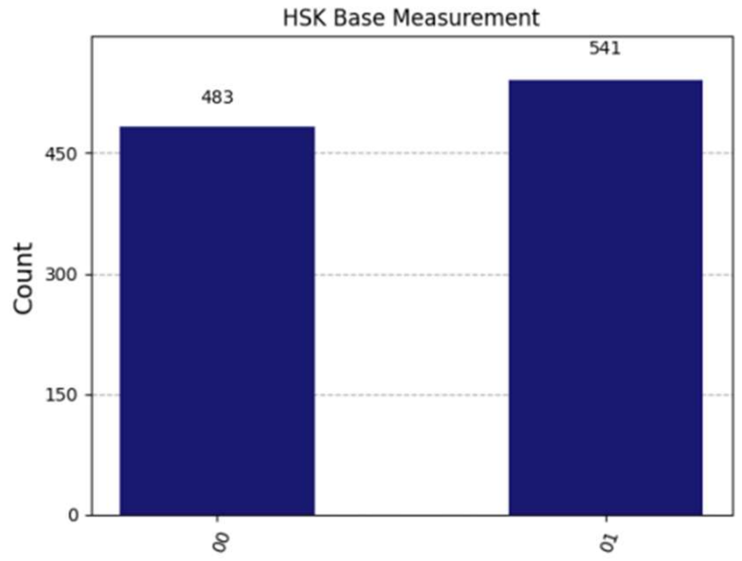


Tensorprodukt Operatoren

Q-Circuit:

$H \otimes I$

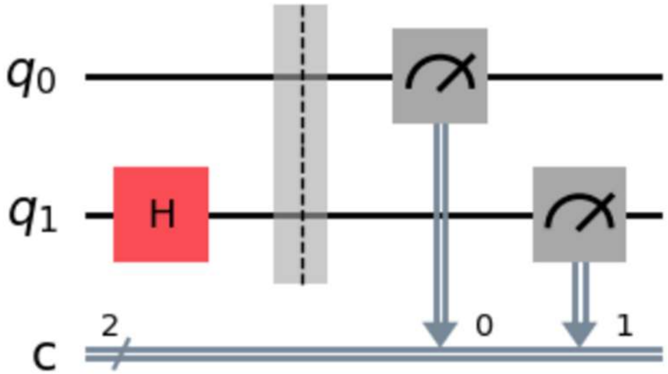


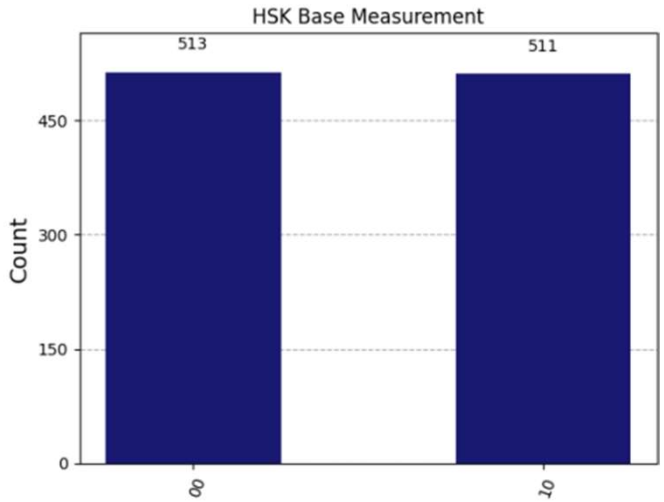


Tensorprodukt Operatoren

Q-Circuit:

$I \otimes H$





Quantenregister Messung

Zustand:

$$|\text{xy}>= \alpha_x\alpha_y|00> +\alpha_x\beta_y|01> +\beta_x\alpha_y|10> +\beta_x\beta_y|11>$$

Messung beide qBits:

|00>	$\left\|\alpha_x\alpha_y\right\|^2$
|01>	$\left\|\alpha_x\beta_y\right\|^2$
|10>	$\left\|\beta_x\alpha_y\right\|^2$
|11>	$\left\|\beta_x\beta_y\right\|^2$

Quantenregister Messung

Zustand:

$$|\mathrm{xy}>= \alpha_x\alpha_y\big|00 > +\alpha_x\beta_y\big|01 > +\beta_x\alpha_y|10 > +\beta_x\beta_y|11 >$$

Messung qBit0:

|00> |01>	$\left\|\alpha_x\alpha_y\right\|^2 + \left\|\alpha_x\beta_y\right\|^2$
|10> |11>	$\left\|\beta_x\alpha_y\right\|^2 + \left\|\beta_x\beta_y\right\|^2$

Quantenregister Messung

Zustand:

$$|\mathrm{xy}>= \alpha_x\alpha_y|00> +\alpha_x\beta_y|01> +\beta_x\alpha_y|10> +\beta_x\beta_y|11>$$

Messung qBit1:

|00> |10> $\qquad \|\alpha_x\alpha_y\|^2 + \|\beta_x\alpha_y\|^2$

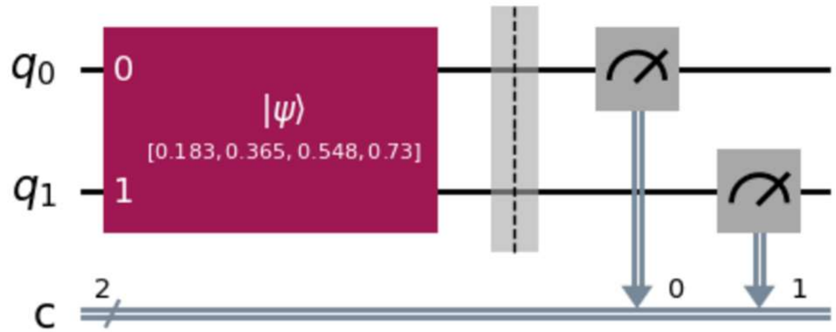
|01> |11> $\qquad \|\alpha_x\beta_y\|^2 + \|\beta_x\beta_y\|^2$

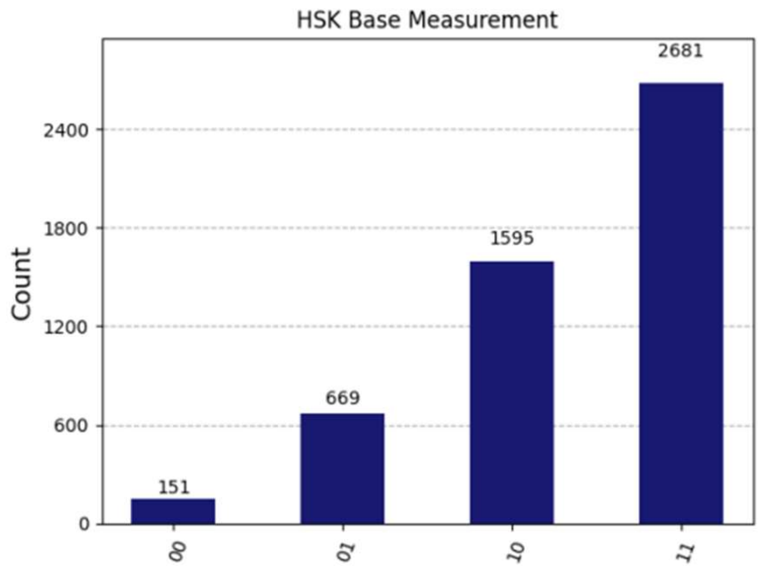
Quantenregister Messung

Zustand:

$|xy>= .183|00> +.365|01> +.548|10> .73|11>$

Messung qBit0 und qBit1:



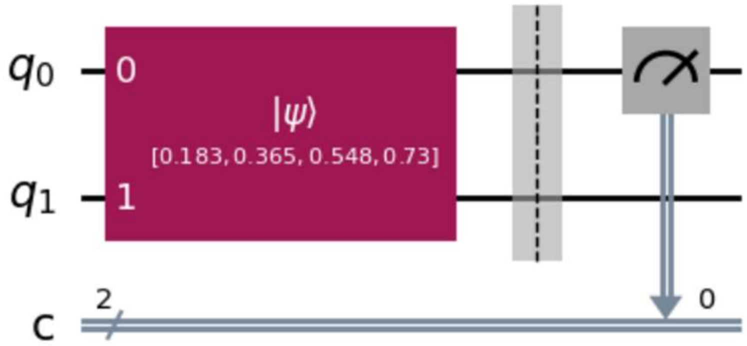


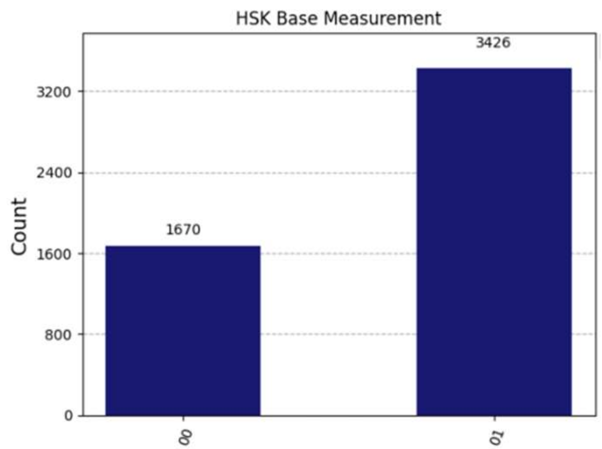
Quantenregister Messung

Zustand:

$|xy>= .183|00> +.365|01> +.548|10> .73|11>$

Messung qBit0:





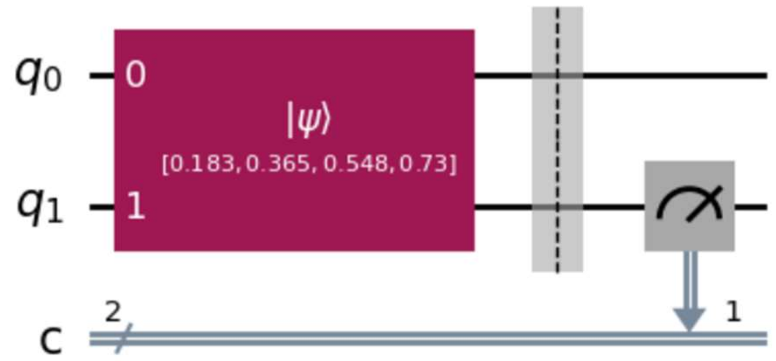
$$\|\alpha_x\alpha_y\|^2 + \|\alpha_x\beta_y\|^2 \approx 5096 \cdot \left((.183)^2 + (.548)^2\right) \approx 1701$$
$$\|\beta_x\alpha_y\|^2 + \|\beta_x\beta_y\|^2 \approx 5096 \cdot \left((.365)^2 + (.73)^2\right) \approx 3394$$

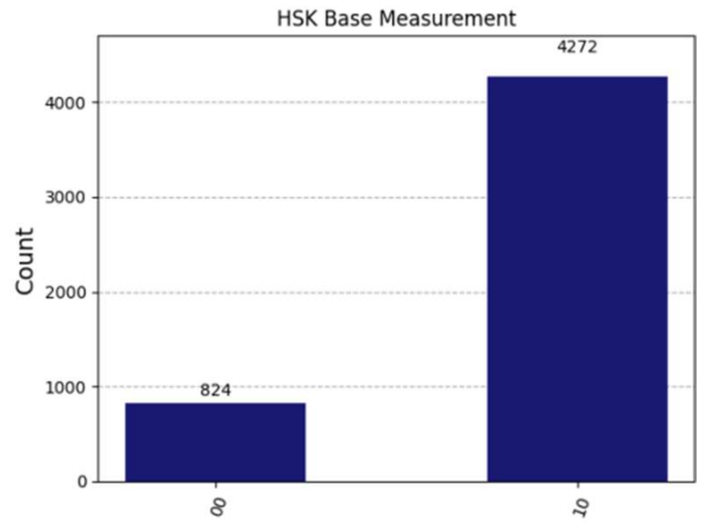
Quantenregister Messung

Zustand:

$|xy> = .183|00> + .365|01> + .548|10> .73|11>$

Messung qBit1:





$$\|\alpha_x \alpha_y\|^2 + \|\beta_x \alpha_y\|^2 \approx 5096 \cdot \left((.183)^2 + (.365)^2\right) \approx 849$$

$$\|\alpha_x \beta_y\|^2 + \|\beta_x \beta_y\|^2 \approx 5096 \cdot \left((.548)^2 + (.73)^2\right) \approx 4246$$

Verschränkung

Bell States:

$$|\phi^{+}>=\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)$$

$$|\phi^{-}>=\frac{1}{\sqrt{2}}(|00\rangle-|11\rangle)$$

$$|\psi^{+}>=\frac{1}{\sqrt{2}}(|01\rangle+|10\rangle)$$

$$|\psi^{-}>=\frac{1}{\sqrt{2}}(|01\rangle-|10\rangle)$$

Concurrence:

$$|\mathrm{xy}>=\alpha_x\alpha_y\big|00>+\alpha_x\beta_y\big|01>+\beta_x\alpha_y|10>+\beta_x\beta_y|11>$$

$$C(|\mathrm{xy}>)=2\cdot\left|\left(\alpha_x\alpha_y\cdot\beta_x\beta_y\right)-\left(\alpha_x\beta_y\cdot\beta_x\alpha_y\right)\right|$$

$$0\leq C(|\mathrm{xy}>)\leq 1$$

Verschränkung

2qBit State:

$$|\mathrm{xy}> = \alpha_x \alpha_y \big|00> + \alpha_x \beta_y \big|01> + \beta_x \alpha_y |10> + \beta_x \beta_y |11>$$

Separierbar:

$$\alpha_x \alpha_y \cdot \beta_x \beta_y = \alpha_x \beta_y \cdot \beta_x \alpha_y \qquad \leftrightarrow \qquad C(|\mathrm{xy}>) = 0$$

$$|xy\rangle = |x\rangle \otimes |y\rangle$$

Verschränkt:

$$\alpha_x \alpha_y \cdot \beta_x \beta_y \neq \beta_y \cdot \beta_x \alpha_y \qquad \leftrightarrow \qquad C(|\mathrm{xy}>) > 0$$

Verschränkung

Allgemeiner verschränkter Zustand:

$$|\mathrm{xy}>= \frac{1}{\sqrt{k}}|00> + \frac{\sqrt{k-1}}{\sqrt{k}}|11>, \; k \geq 2$$

Concurrence:

$$C_k(|\mathrm{xy}>) = 2 \cdot \left(\frac{\sqrt{k-1}}{k}\right) \qquad \lim_{k \to \infty} C_k = 0$$

Invarianz von C:

$$C(|\mathrm{xy}>) = C(|\mathrm{x}(Uy)>) = C(|(\mathrm{Ux})\mathrm{y}>)$$