Einführung

Quantum Computing

HSK 9.10.2024

- **Modifikation von qBits**
  - **Unitäre Transformationen**

Skalarprodukt/Norm

Vektorraum $\mathbb{C}^{\boldsymbol{n}}$

Vektoren $\vec{a}, \vec{b}$ ε $\mathbb{C}^{\boldsymbol{n}}$:

$$\vec{a} = \begin{pmatrix} a_1 \\ \cdots \\ a_n \end{pmatrix}, \vec{b} = \begin{pmatrix} b_1 \\ \cdots \\ b_n \end{pmatrix}$$

Skalarprodukt:

$$\langle \vec{a}, \vec{b} \rangle = a_1^* \, b_1 + \cdots + a_n^* b_n$$

Norm:

$$\|\vec{a}\| = \sqrt{\langle \vec{a}, \vec{a} \rangle} = \sqrt{a_1^* a_1 + \cdots + a_n^* a_n} = \sqrt{|a_1|^2 + \cdots + |a_n|^2}$$

qBit:

$$|x\rangle = \alpha \binom{1}{0} + \beta \binom{0}{1} = \binom{\alpha}{\beta} \qquad \||x\rangle\| = \sqrt{\alpha^* \cdot \alpha + \beta^* \cdot \beta} = 1$$

Matrix mal Vektor

Vektorraum $\mathbb{C}^{\boldsymbol{n}}$

Matrix A Ɛ $\mathbb{C}^{\boldsymbol{n}\times\boldsymbol{n}}$:

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix}$$

Produkt:

$$A|x\rangle == \begin{pmatrix} a_{11} \cdot x_1 & + & a_{1n} \cdot x_n \\ \vdots & \ddots & \vdots \\ a_{n1} \cdot x_1 & + & a_{nn} \cdot x_n \end{pmatrix}$$

qBit:

$$|x\rangle = \alpha\binom{1}{0} + \beta\binom{0}{1} = \binom{\alpha}{\beta} \qquad \mathrm{A}|x\rangle = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\binom{\alpha}{\beta} = \begin{pmatrix} \alpha \cdot a_{11} + \beta \cdot a_{12} \\ \alpha \cdot a_{21} + \beta \cdot a_{22} \end{pmatrix}$$

Unitäre Matrizen

Vektorraum $\mathbb{C}^{\boldsymbol{n}}$

Matrix A Ɛ $\mathbb{C}^{\boldsymbol{n}\times\boldsymbol{n}}$:

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix}$$

Adjungierte Matrix:

$$A^{\dagger} = (A^{*})^{T}$$

Unitäre Matrix:

$$U^{\dagger} = U^{-1}$$

qBit:

$$|x\rangle = \alpha\binom{1}{0} + \beta\binom{0}{1} = \binom{\alpha}{\beta} \qquad \| U|x\rangle \| = \| |x\rangle \| = 1$$

Quanten Gatter

Identität:

$$\mathrm{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Pauli:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

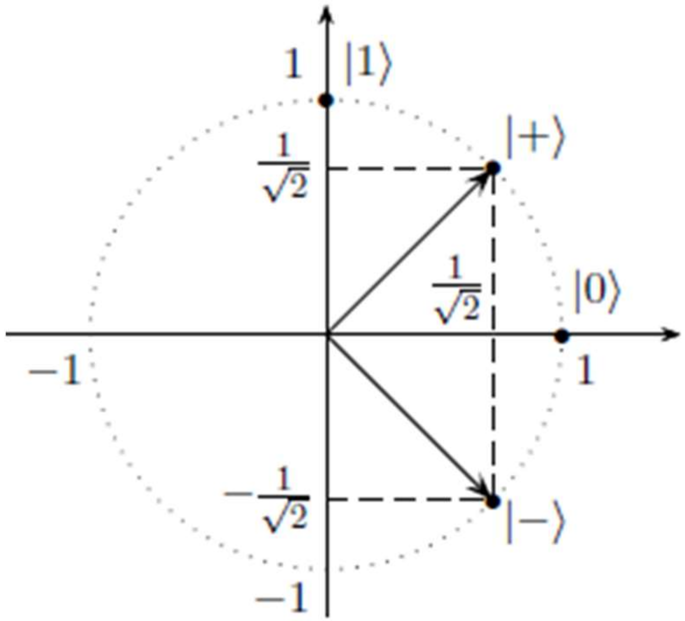
Hadamard:

$$H = \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Hadamard Gate

$H|0\rangle = |+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

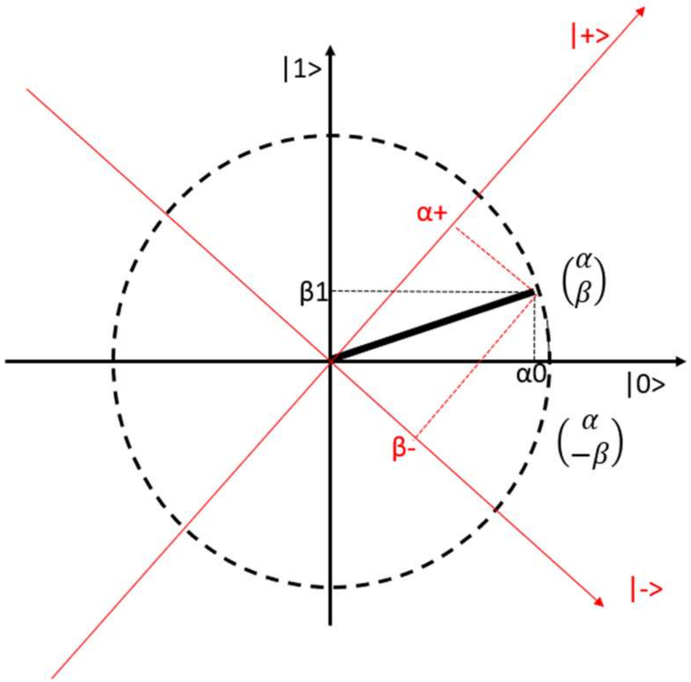
$H|1\rangle = |-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$



Hadamard Basis

$H|0\rangle = |+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

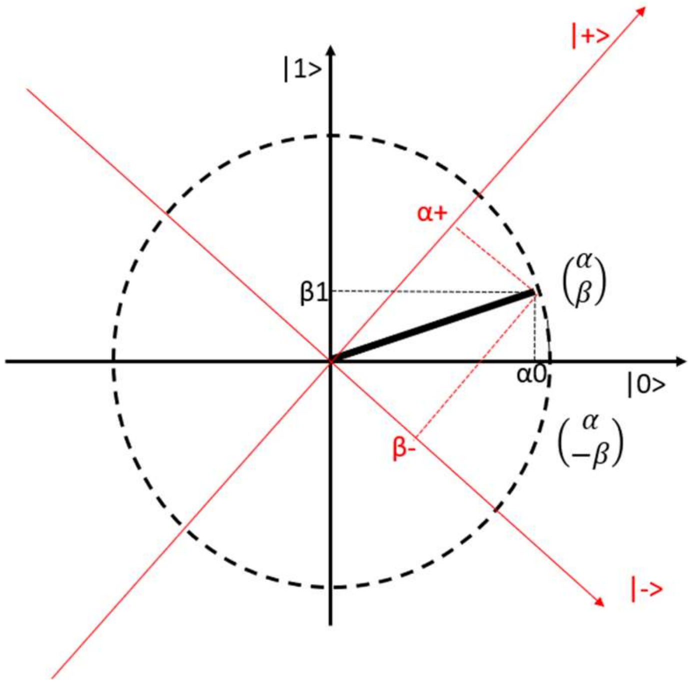
$H|1\rangle = |-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$

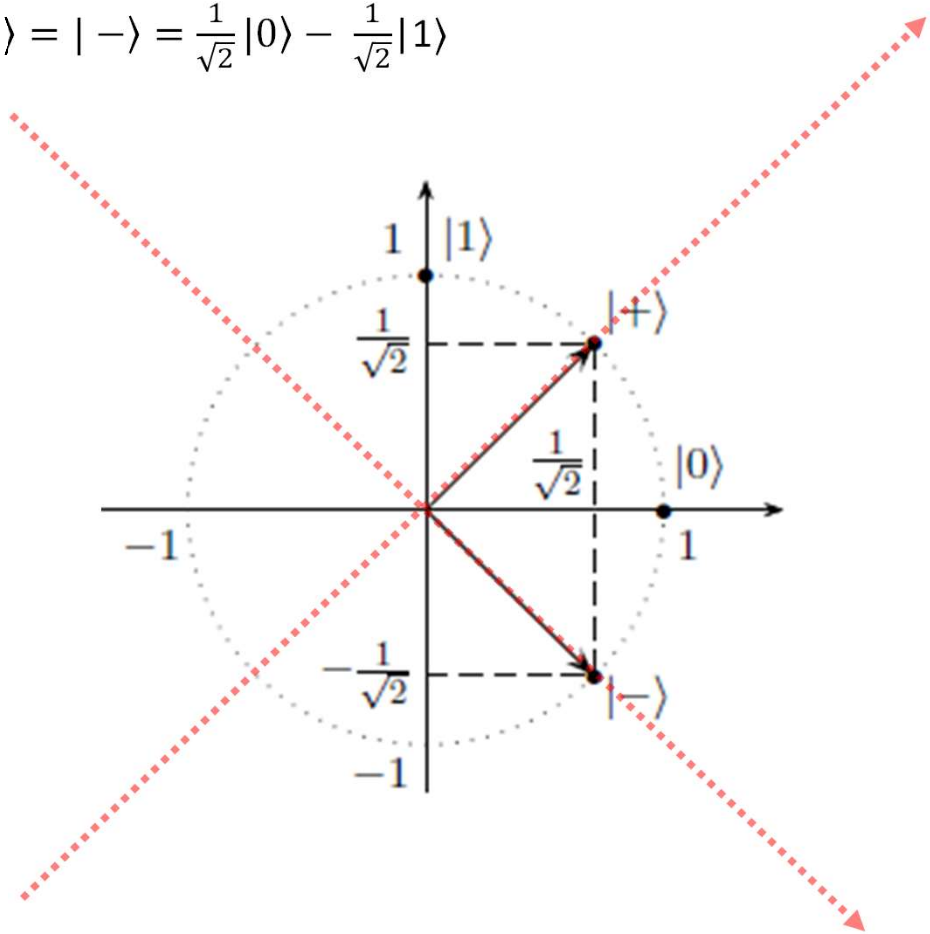


Hadamard Basis

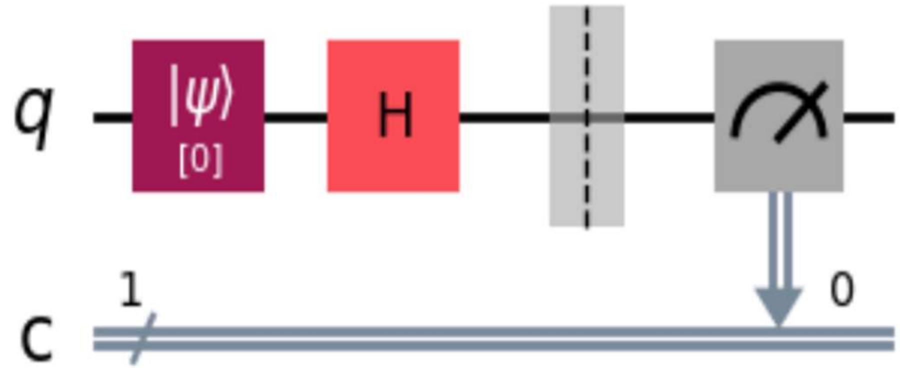
$H|0\rangle = |+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

$H|1\rangle = |-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$



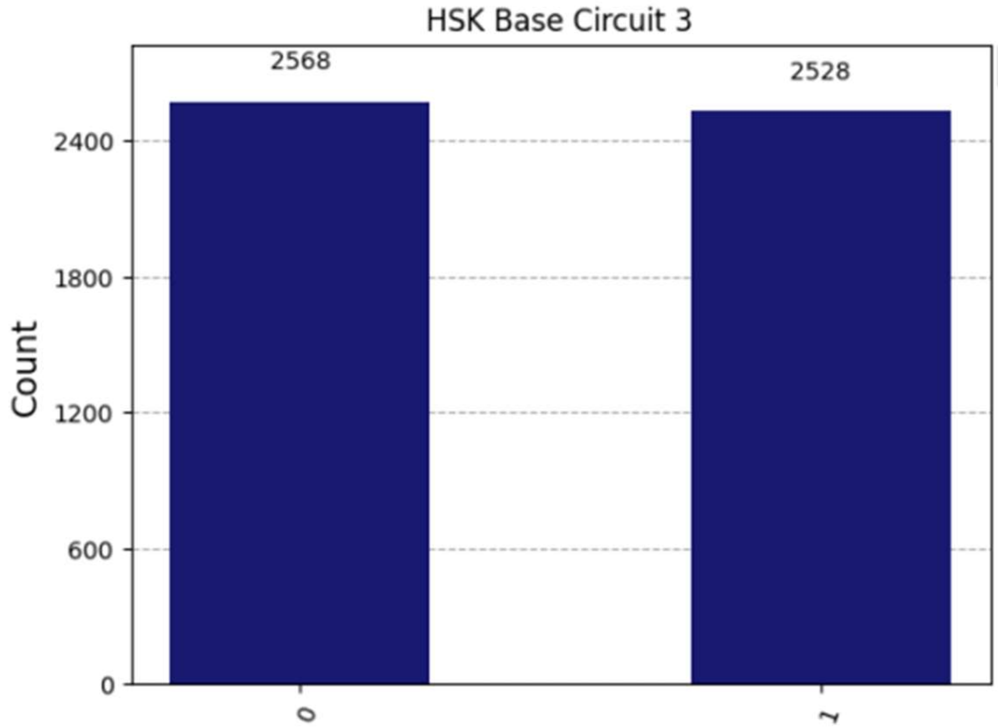


Zufallsgenerator



$$H|0\rangle = |+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

Zufallsgenerator



$$H|0\rangle = |+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$