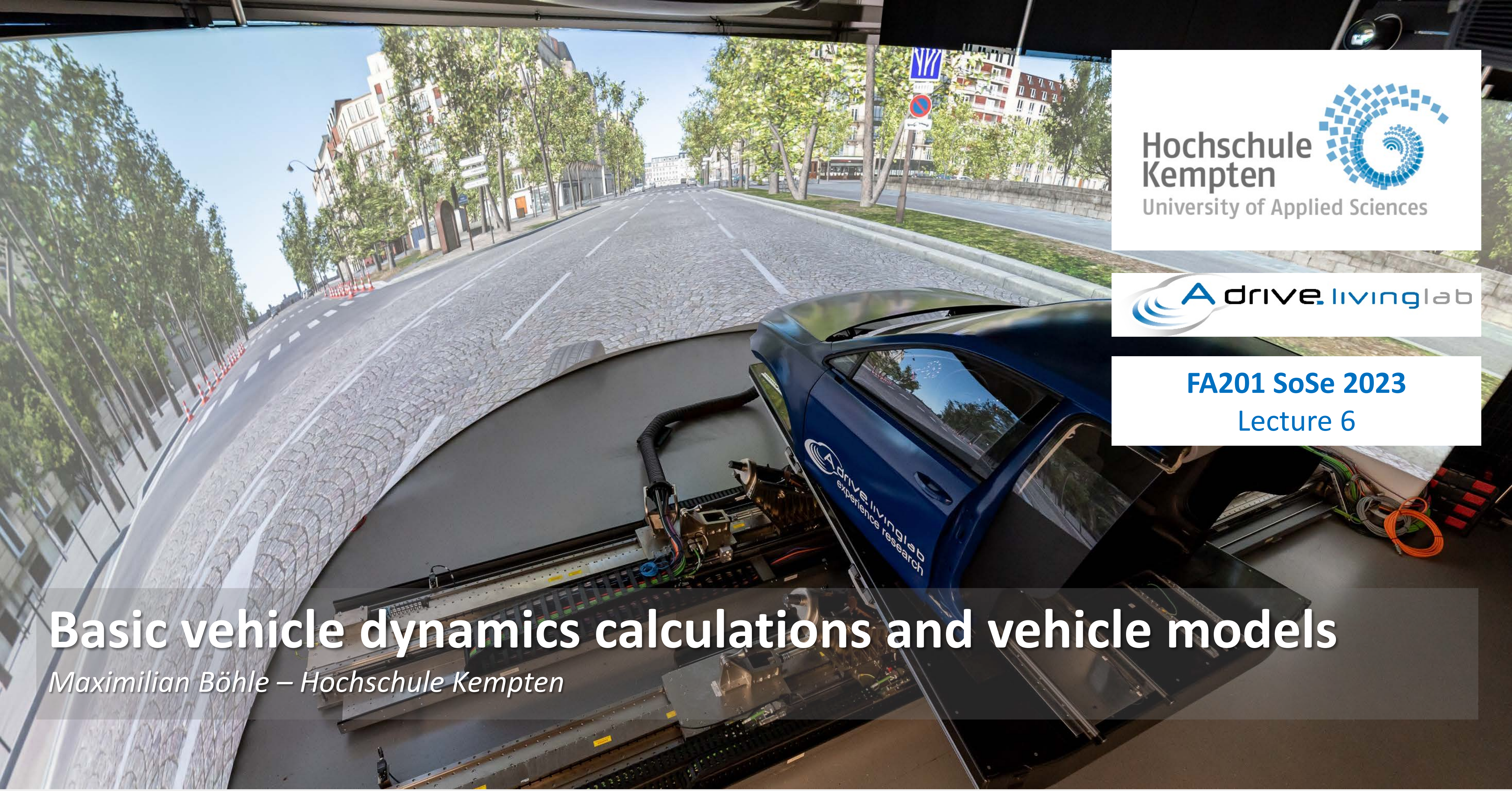
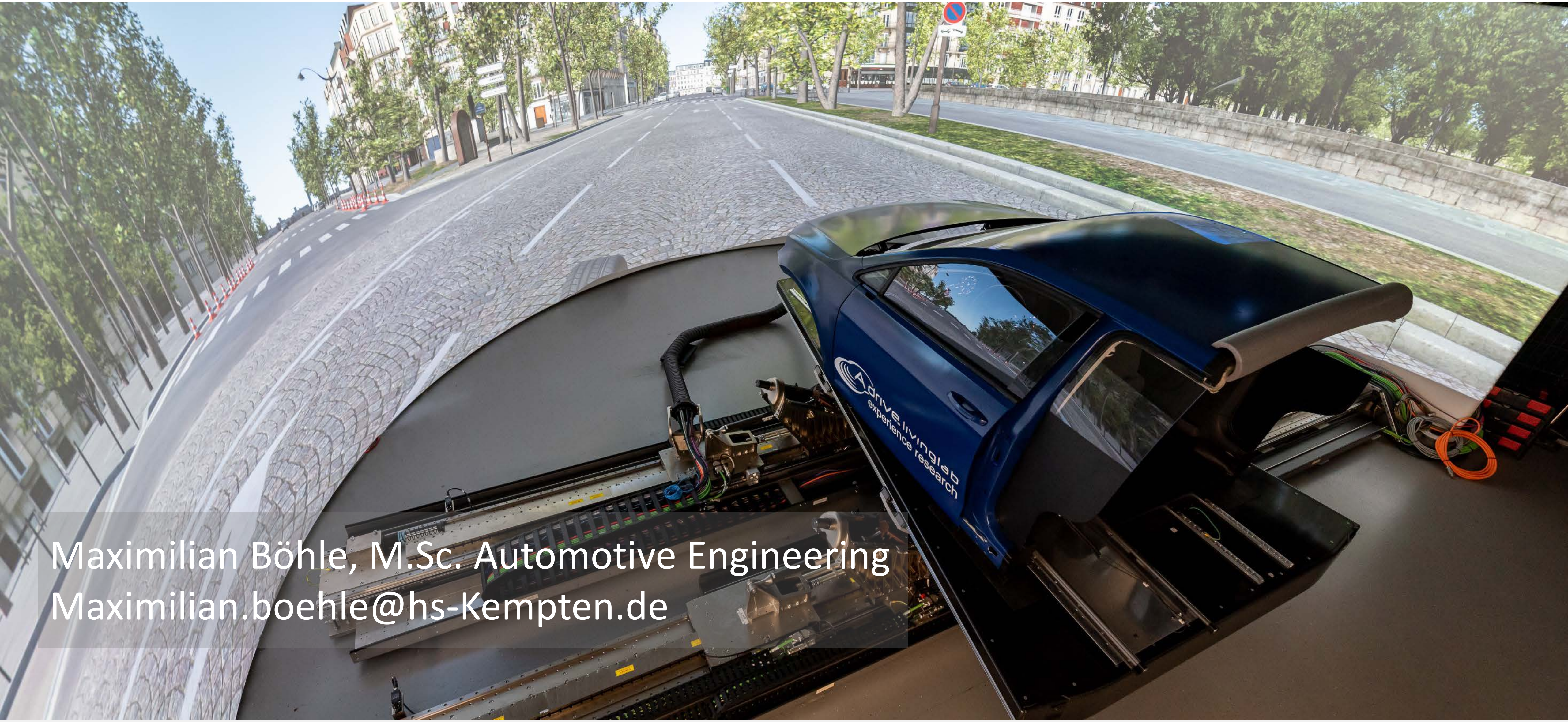


FA201 SoSe 2023
Lecture 6

Basic vehicle dynamics calculations and vehicle models

Maximilian Böhle – Hochschule Kempten

Basic vehicle dynamics calculation and vehicle models



Maximilian Böhle, M.Sc. Automotive Engineering
Maximilian.boehle@hs-Kempten.de

Agenda

- Basics
  - Forces, moments, masses
  - Equations of motion
  - Recap/Preview: Tires

- Basic calculations – dynamic
  - Self-steering behaviour
  - Practice session

- Validity and limitations
- Applications

Recommended literature

- Mitschke, Manfred, and Henning Wallentowitz. "Dynamik der Kraftfahrzeuge. 5., überarb. u. erg. Auflage." (2014).

→ Chapter 20: Lineares Einspurmodell, objektive Kenngrößen, Subjektivurteile
→ Chapter 21: Kreisfahrt bei konstanter Fahrgeschwindigkeit

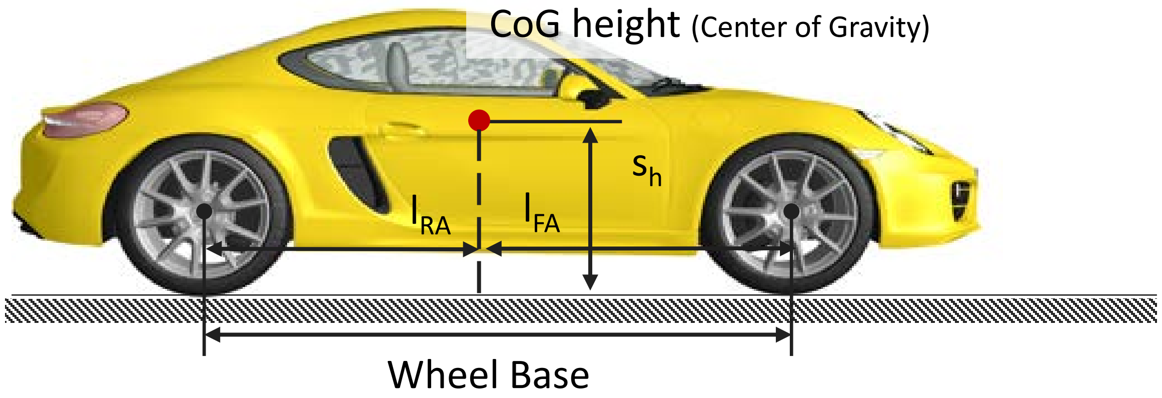
- Ersoy, Metin, and Stefan Gies, eds. *Fahrwerkhandbuch: Grundlagen–Fahrdynamik–Fahrverhalten–Komponenten –Elektronische Systeme–Fahrerassistenz–Autonomes Fahren–Perspektiven*. Springer-Verlag, 2017.

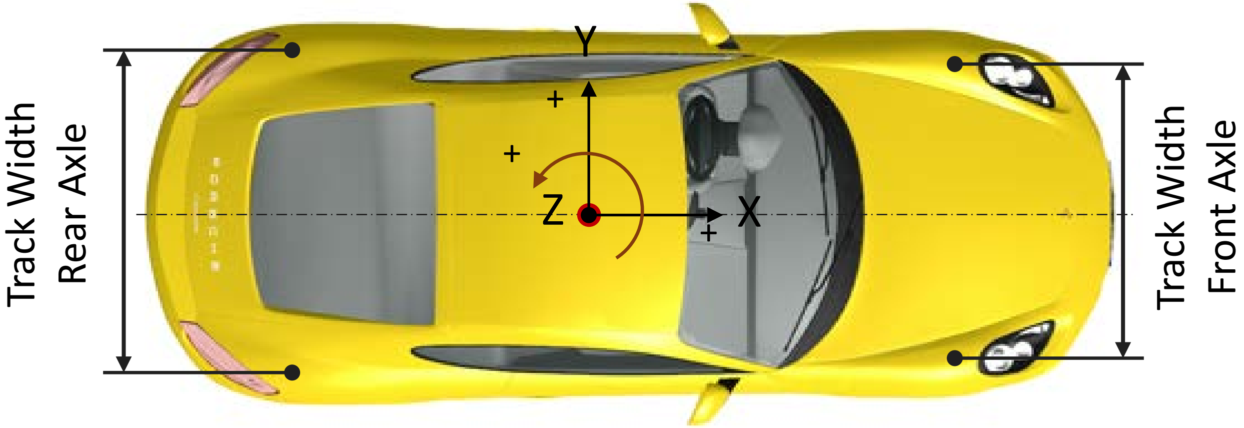
→ Chapter 2: Fahrdynamik

Basic vehicle dynamics calculation and vehicle models

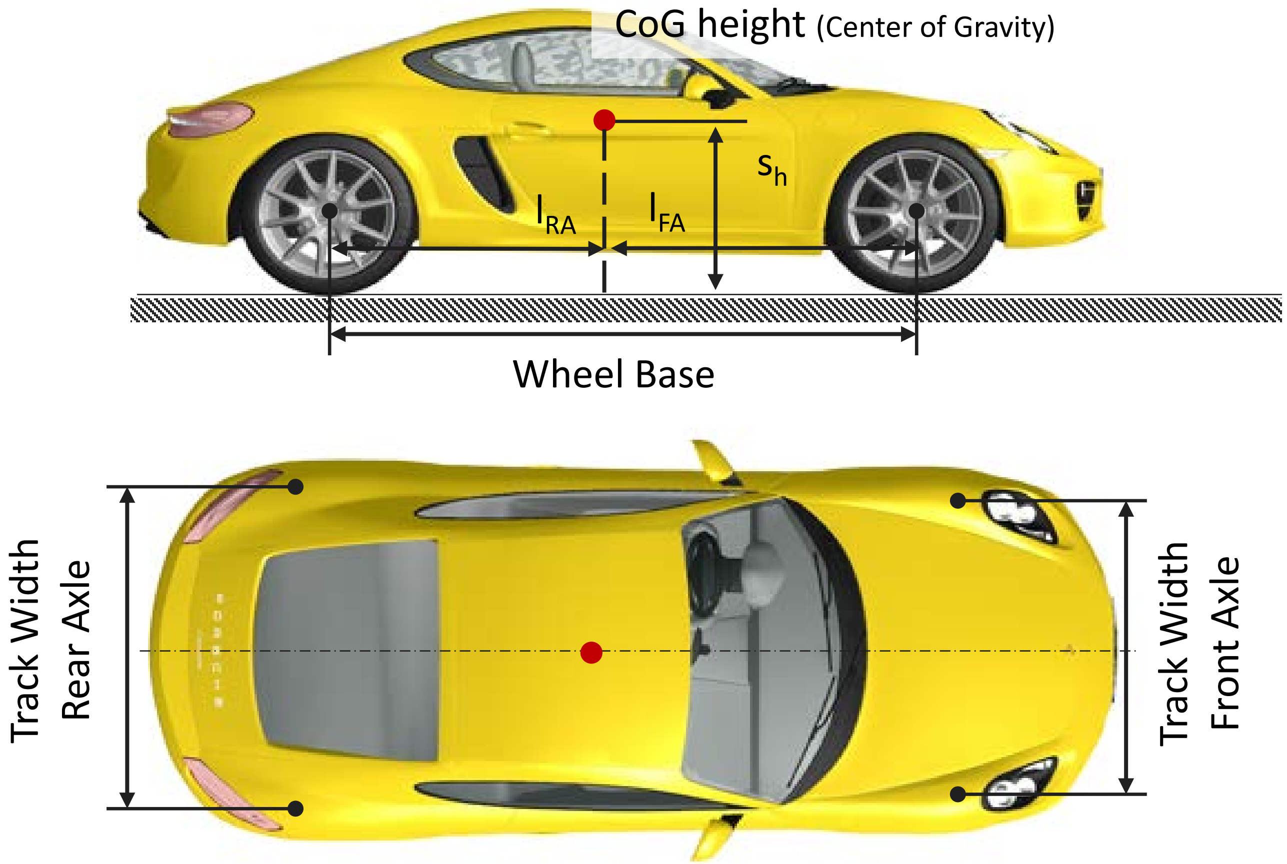
What is a model?		A simplified representation of the reality.
What is a vehicle dynamics model?		A tool to calculate the dynamic motion of ground vehicles for engineering tasks.
Where do we need vehicle dynamics models?		Simulation, model based testing and model based control methods.
When is a vehicle dynamics model valid?		Good and accurate enough for the application purpose?

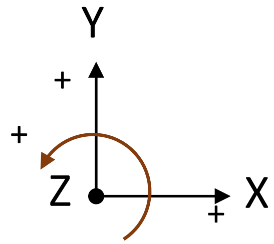
Basic chassis dimensions



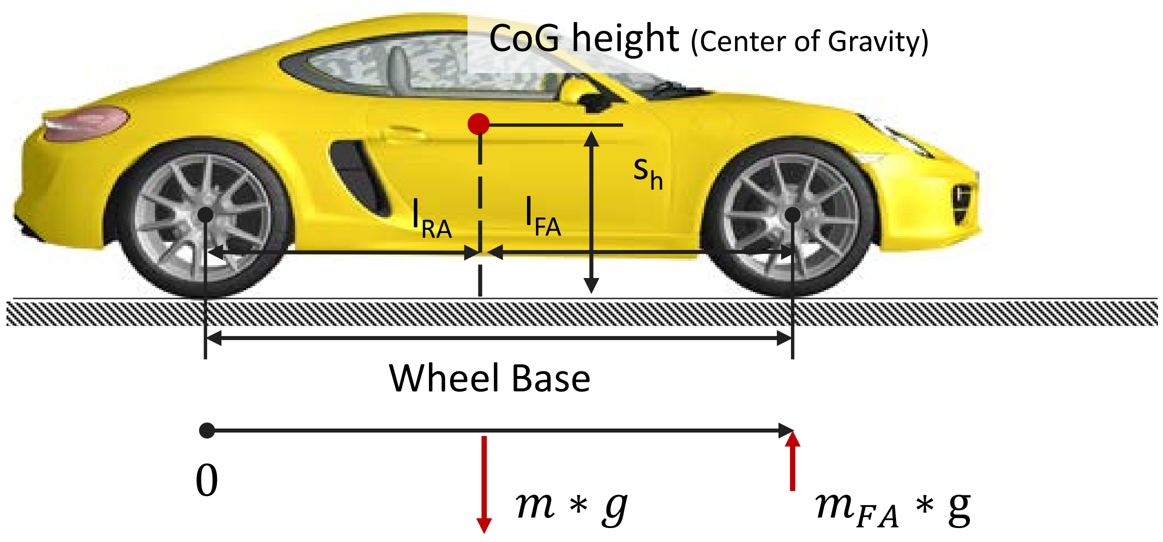


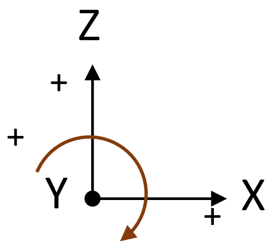
Basic chassis dimensions





Calculation of CoG – Center of Gravity





$$\sum F_z = 0$$

$$m * g * l_{RA} = m_{FA} * g * l$$

$$l_{FA} = l - l_{RA}$$

- Vehicle weight (with driver) = 1.970 kg
- m_{FA} (front axle) = 1100 kg
- m_{RA} (rear axle) = 870 kg
- Wheel base = 2.807 mm,
- Center of gravity = 0,65 m

Calculation of CoG – Center of Gravity

$$m * \cancel{g} * l_{RA} = m_{FA} * \cancel{g} * l$$

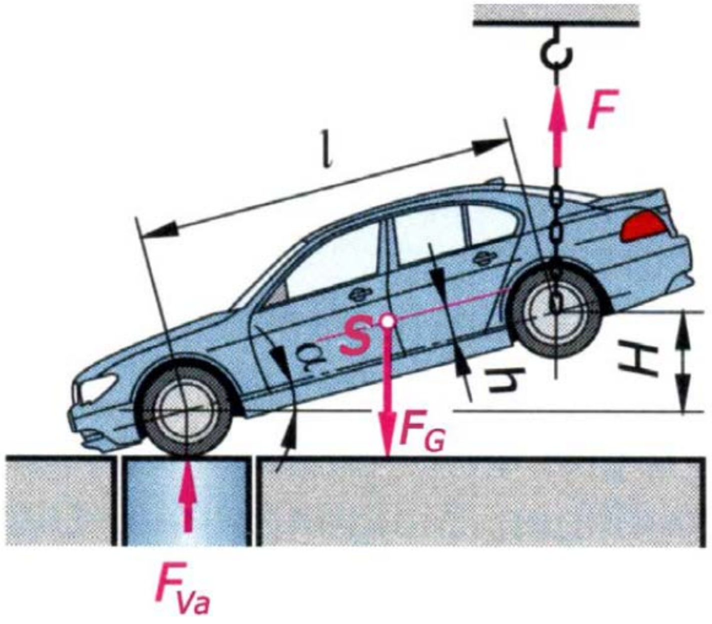
$$\boldsymbol{l_{RA}} = \frac{m_{FA} * l}{m} = \frac{1100\,kg * 2{,}807\,m}{1970\,kg} = \mathbf{1{,}567\,m}$$

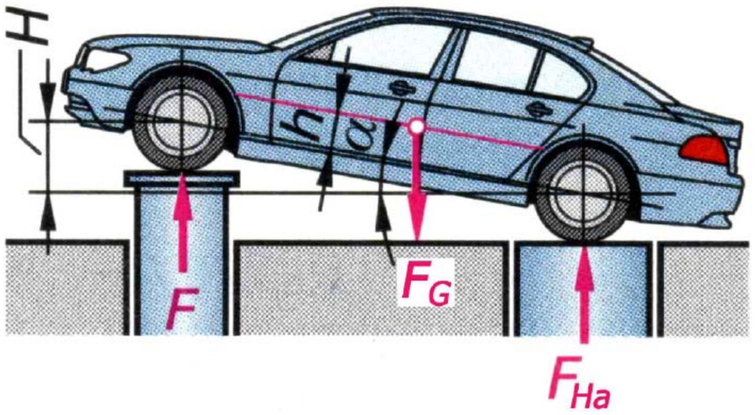
$$l_{FA} = l - l_{RA}$$

$$\boldsymbol{l_{FA}} = l - l_{RA} = 2{,}807m - 1{,}567m = 1{,}24\,\boldsymbol{m}$$

- Vehicle weight (with driver) = 1.970 kg
- m_{FA} (front axle) = 1100 kg
- m_{RA} (rear axle) = 870 kg
- Wheel base = 2.807 mm,
- Center of gravity = 0,65 m

Calculation of CoG – Center of Gravity





[1]

$$h = l * \frac{F_{FA} - F}{F_G * tan\alpha}$$

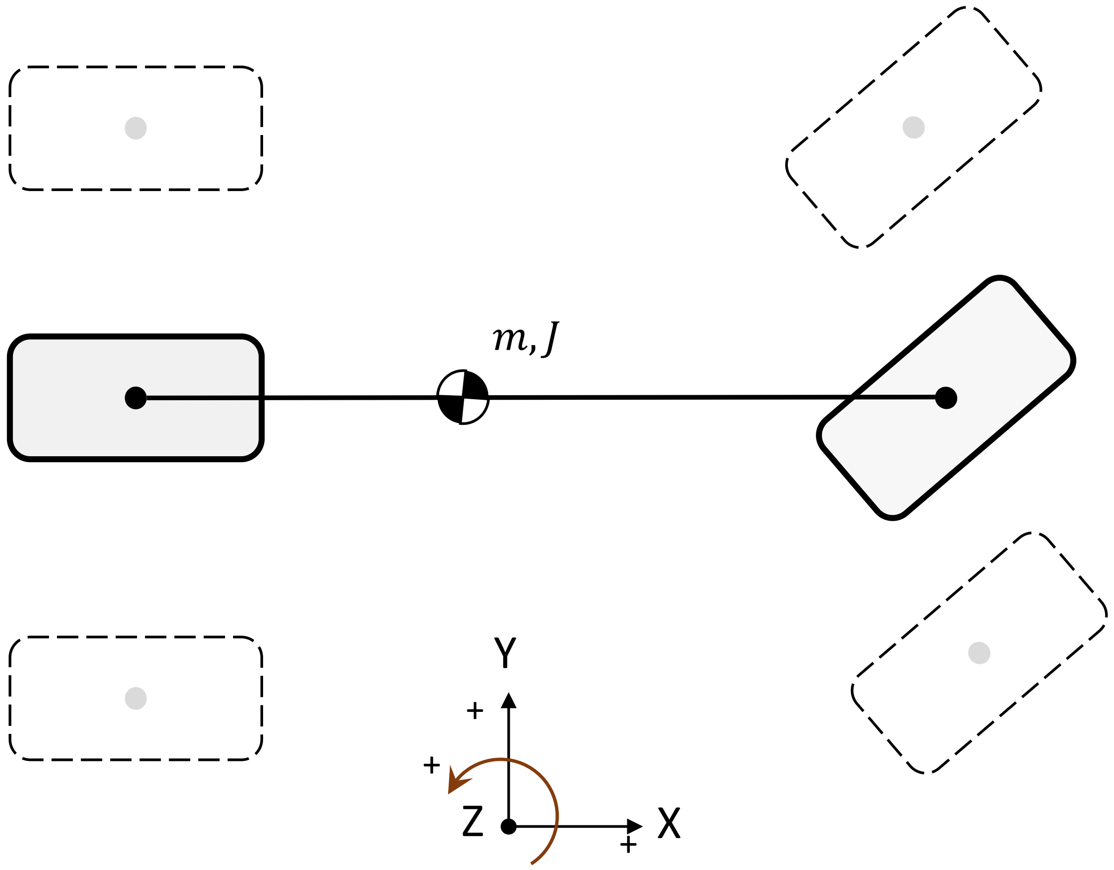
$$h = l * \frac{F_{HA} - F}{F_G * tan\alpha}$$

Calculation of CoG – Center of Gravity

[2]

[3]

Simplification of the vehicle model: "Single Track Model" Theory - 3 DOF



Approach

- **Wheels are lumped into single track**
  → Tire side slip & axle stiffness are combined per axle

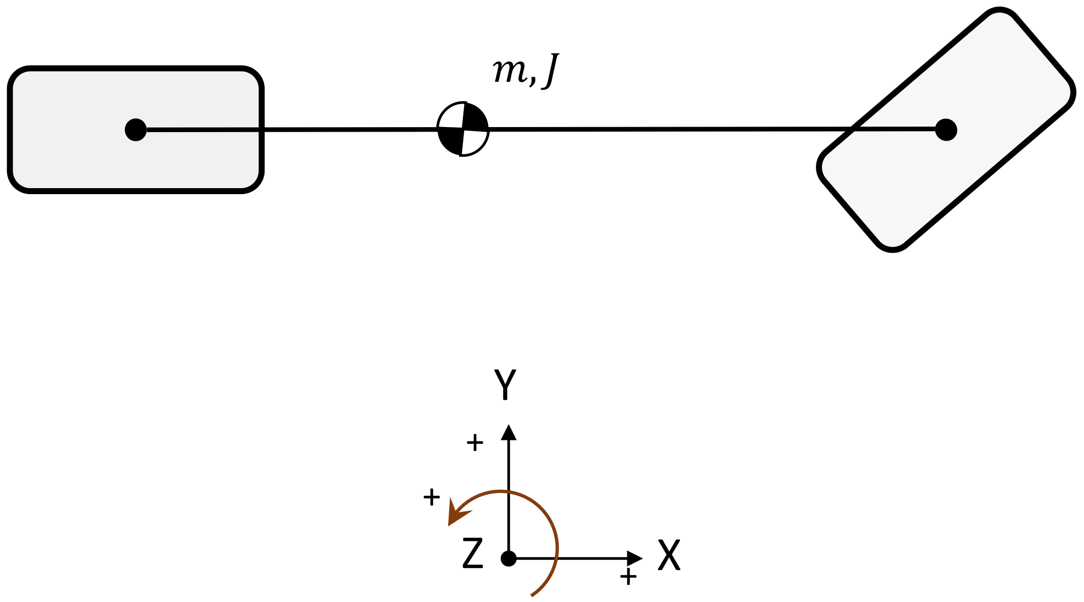
- Rigid body with CoG in-plane (on-track)
- Only horizontal movement
  → No roll, pitch & vertical motion

- Steering angle only at the front axle

3 Degrees of Freedom (DOF)

- Longitudinal
- Lateral
- Yaw (rotation around Z)

Simplification of the vehicle model: “Single Track Model” Theory - 3 DOF



Approach

- **Wheels are lumped into single track**
  → Tire side slip & axle stiffness are combined per axle

- Equations of motion based on:
  - Geometrical equation
  - Equilibrium of forces & moments
  - Transversal system stiffness (Tire / Axle)

3 Degrees of Freedom (DOF)

- Longitudinal
- Lateral
- Yaw (rotation around Z)

Notations

m	vehicle mass
F_x, F_y	forces
F_{x_w}, F_{y_w}	wheel forces
v	vehicle speed
v_x, v_y	longitudinal / lateral vehicle velocity
a_x, a_y	longitudinal / lateral vehicle acceleration
v_{FA}, v_{RA}	velocity front / rear axle
l	wheelbase
l_{FA}, l_{RA}	front / rear axle distance to center of gravity
α_{FA}, α_{RA}	slip angle front / rear axle
c_{FA}, c_{RA}	cornering stiffness front / rear axle
β	side slip angle
$\dot{\beta}$	side slip angle velocity

δ	steering angle (at road wheel)
δ_H	steering wheel angle (SWA, at hand wheel)
i_s	steering ratio
ψ	yaw angle
$\dot{\psi}$	yaw angle speed
R, r	course radius
FA, RA	Index front axle, rear axle
$stat$	index for stationary
CoG	center of gravity
EG	understeer gradient (Eigenlenkgradient)

Lateral Motion (1)

$$F_y = F_{y,FA} + F_{y,RA} = m * a_y = m * v * (\dot{\psi} - \dot{\beta})$$

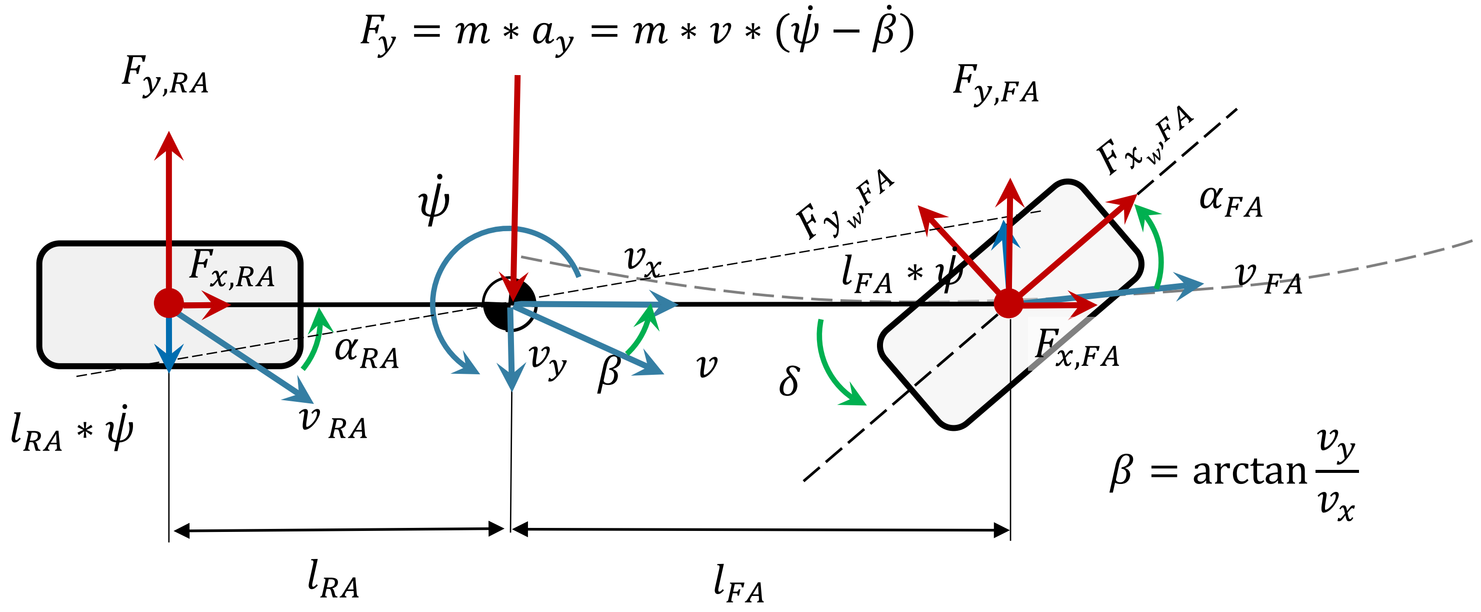
$$\sum F_y = 0$$

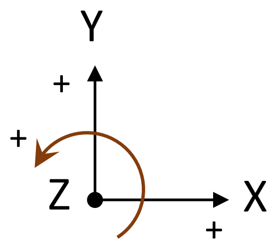
Longitudinal Motion (2)

$$F_x = F_{x,FA} + F_{x,RA} = m * a_x$$

vgl.

$$\sum F_x = 0$$





Rotation Z-Axis (3)

$$\theta * \ddot{\psi} = F_{y,FA} * l_{FA} - F_{y,RA} * l_{RA}$$

$$\sum M_z = 0$$

Basic vehicle dynamics calculation and vehicle models

Lateral Motion (1)

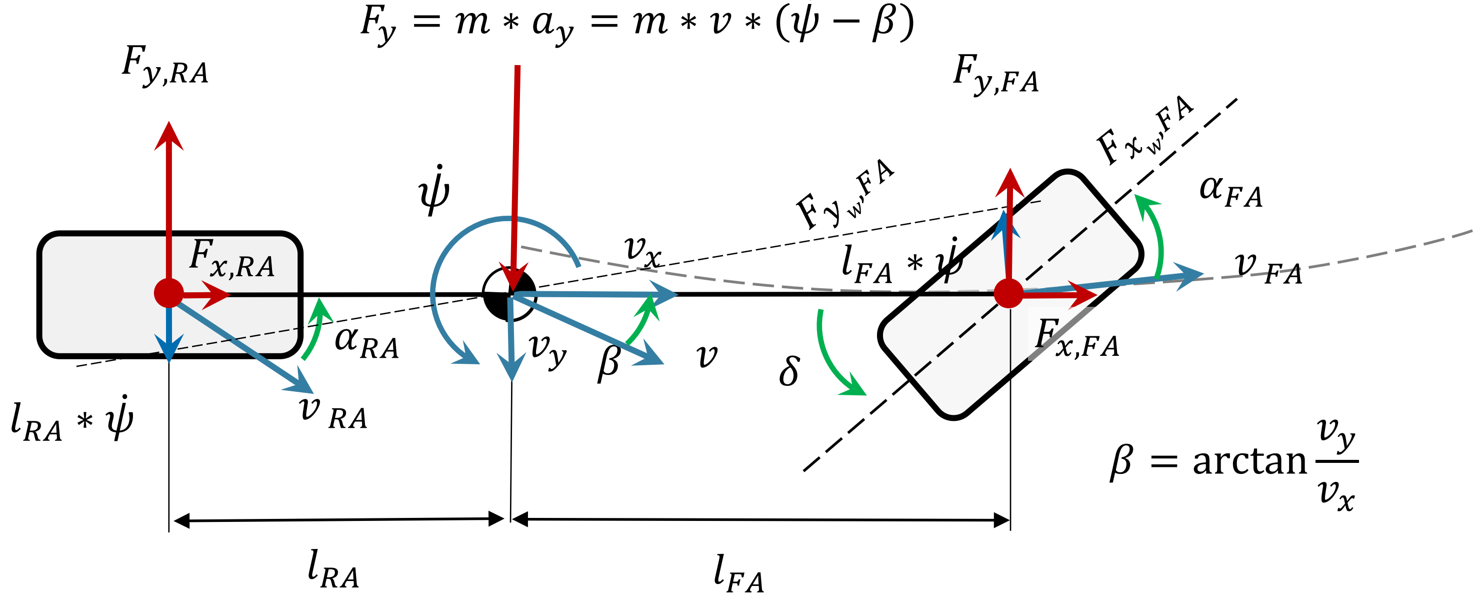
$$F_y = F_{y,FA} + F_{y,RA} = m * a_y = m * v * (\dot{\psi} - \dot{\beta})$$

$$\sum F_y = 0$$

Longitudinal Motion (2)

$$F_x = F_{x,FA} + F_{x,RA} = m * a_x$$

$$\sum F_x = 0$$



Rotation Z-Axis (3)

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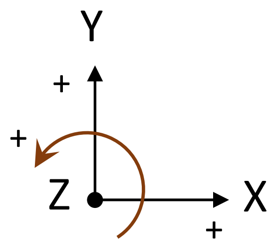


Basic vehicle dynamics calculation and vehicle models

Lateral Motion

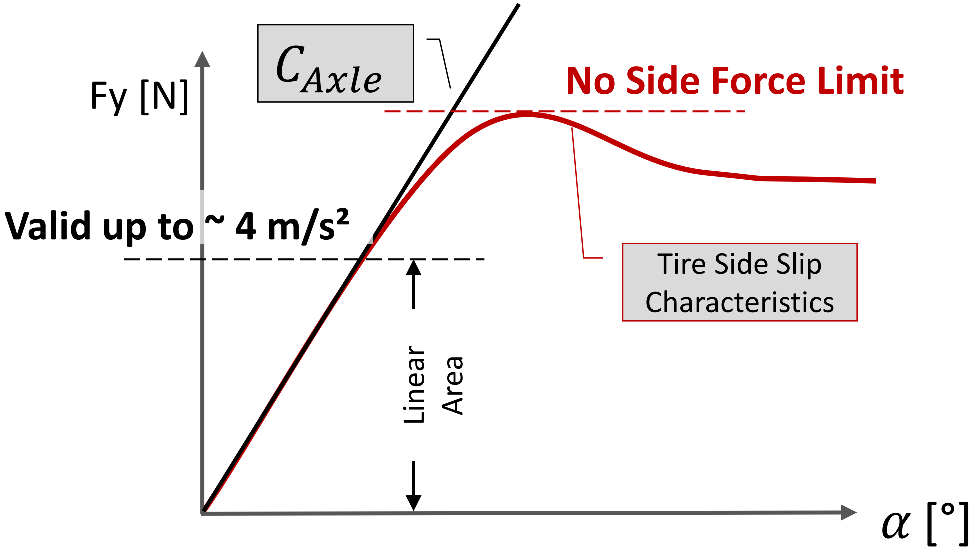
(1) $F_y = F_{y,FA} + F_{y,RA} = m * a_y = m * v * (\dot{\psi} - \dot{\beta})$

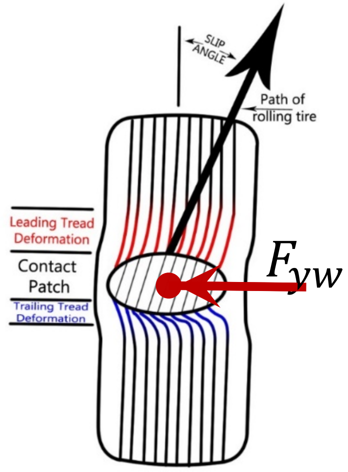
(4) $F_{y,FA} = c_{FA} * \alpha_{FA}$

(5) $\alpha_{FA} = \delta + \beta - \frac{l_{FA} * \dot{\psi}}{v}$

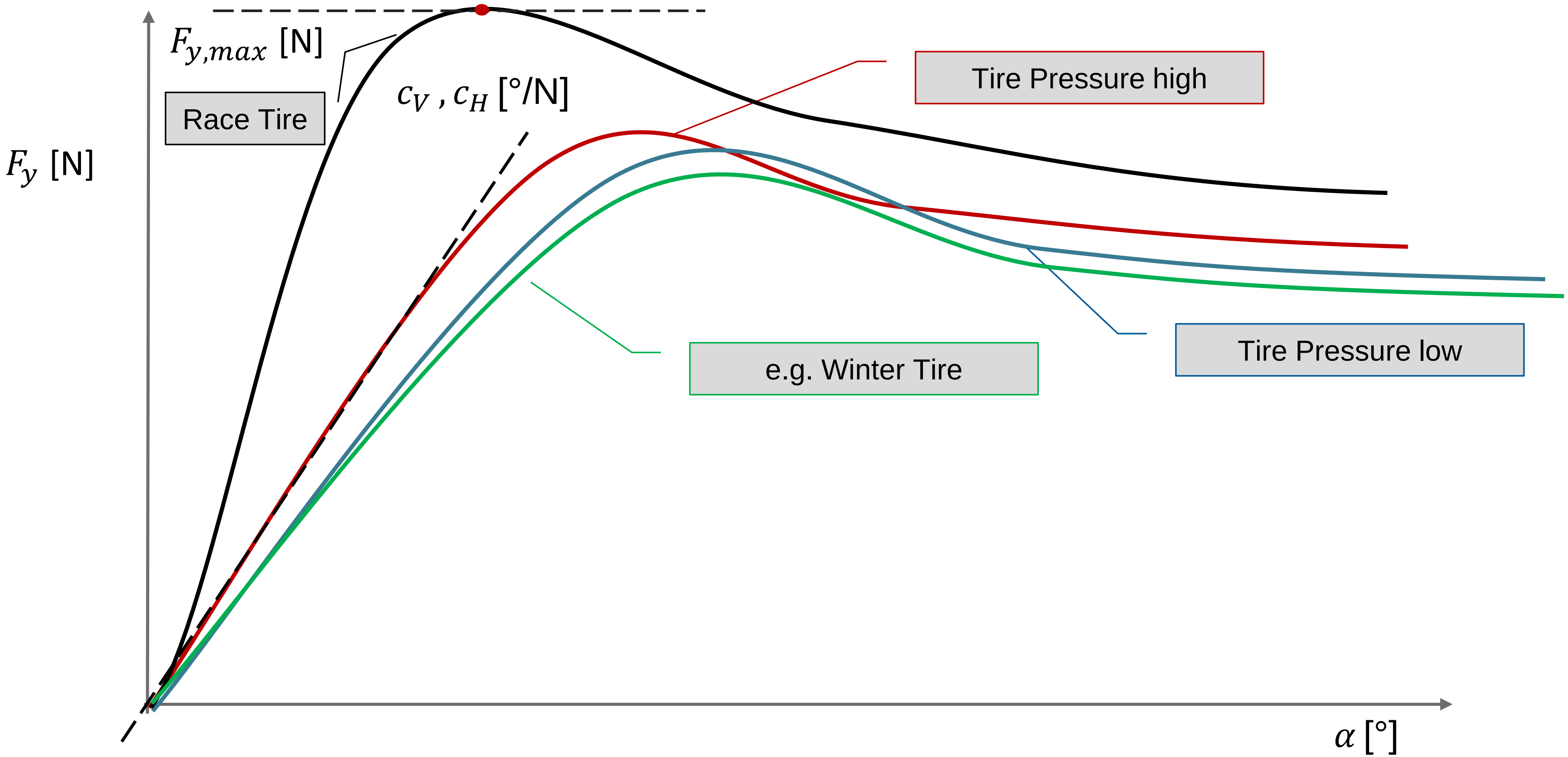
(4) $F_{y,RA} = c_{RA} * \alpha_{RA}$

(5) $\alpha_{RA} = \beta + \frac{l_{RA} * \dot{\psi}}{v}$

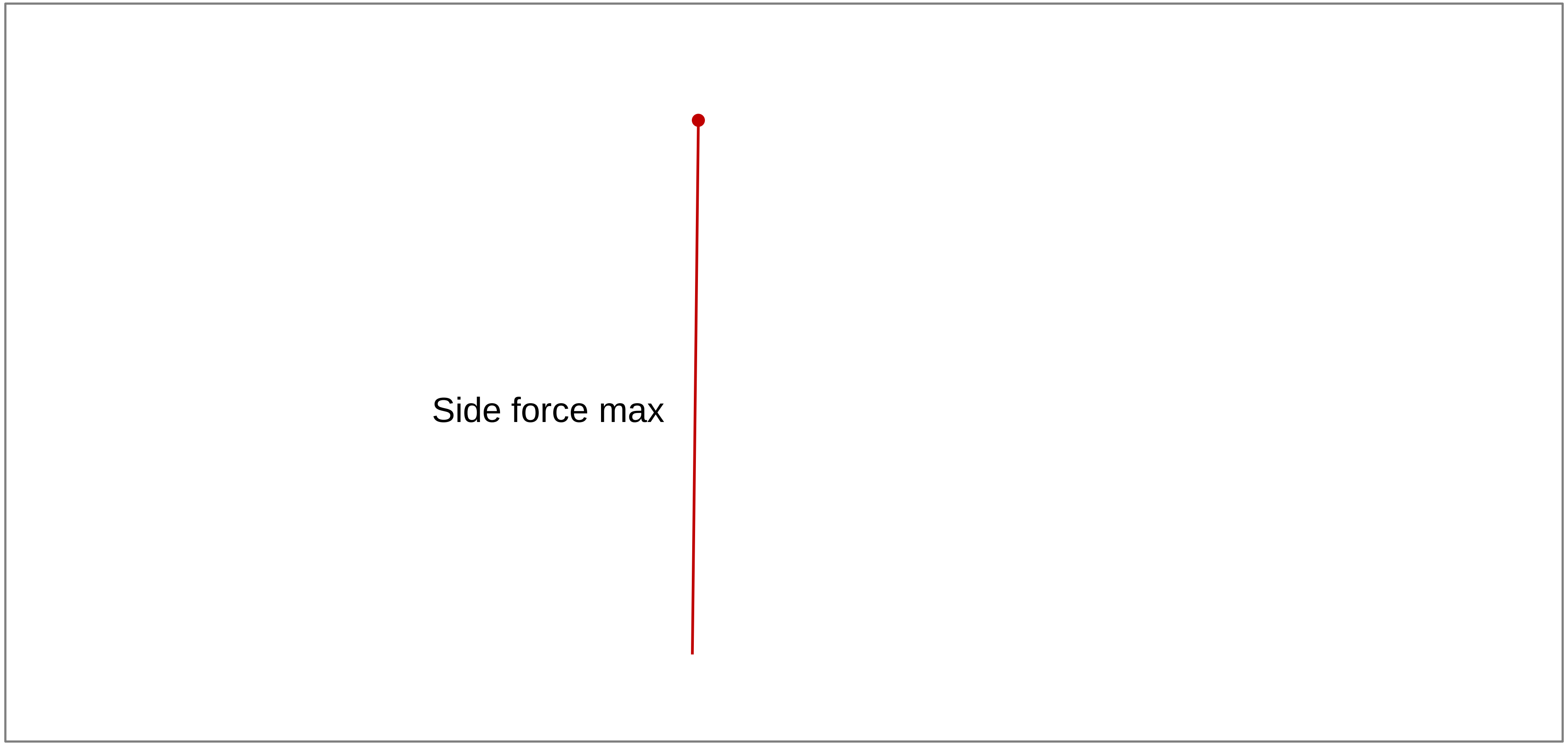




Tire characteristics for understeer behavior (qualitative)



Tire lateral characteristics: side slip behavior



Basic vehicle dynamics calculation and vehicle models

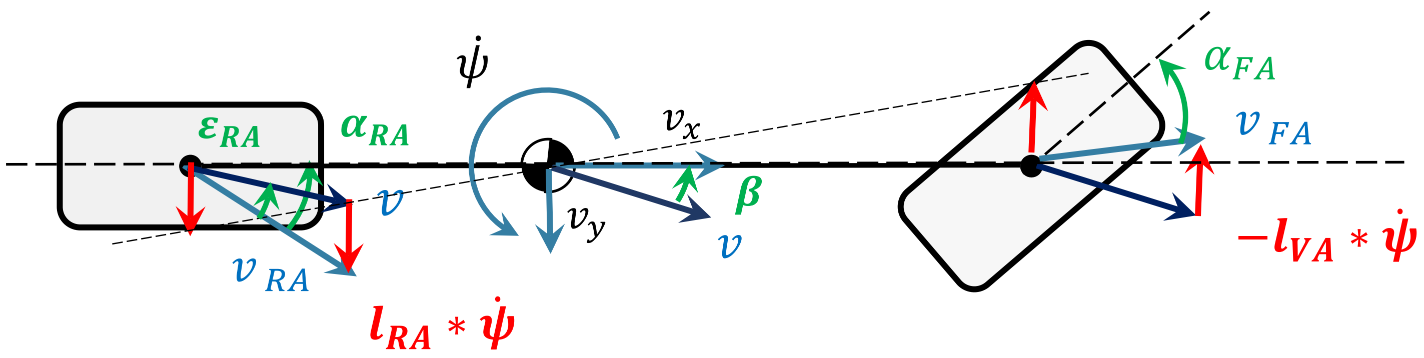
(5)
$$\alpha_{RA} = \beta + \boxed{\frac{l_{RA} * \dot{\psi}}{v}}$$

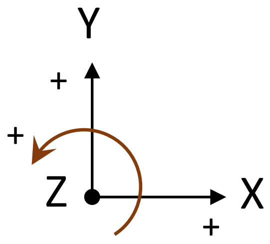
(5)
$$\alpha_{FA} = \delta + \beta \boxed{- \frac{l_{FA} * \dot{\psi}}{v}}$$

Small angle approximation

$$\epsilon_{RA} = \arctan \frac{l_{RA} * \dot{\psi}}{v} \sim \boxed{\frac{l_{RA} * \dot{\psi}}{v}}$$

$$\epsilon_{FA} = -\arctan \frac{l_{FA} * \dot{\psi}}{v} \sim \boxed{- \frac{l_{FA} * \dot{\psi}}{v}}$$





Basic vehicle dynamics calculation and vehicle models

(9) $$c_{FA} * \alpha_{FA} = m * a_y * \frac{l_{RA}}{l}$$

(10) $$c_{RA} * \alpha_{RA} = m * a_y * \frac{l_{FA}}{l}$$

with 5

with 5

(11) $$c_{FA}\left(\delta + \beta - \frac{l_{FA} * \dot{\psi}}{v}\right) = m * a_y * \frac{l_{RA}}{l}$$

(12) $$c_{RA} * \left(\beta + \frac{l_{RA} * \dot{\psi}}{v}\right) = m * a_y * \frac{l_{FA}}{l}$$

(13) $$\beta = \frac{m}{l} * a_y * \frac{l_{RA}}{c_{FA}} + \frac{l_{FA} * \dot{\psi}}{v} - \delta$$

(14) $$\beta = \frac{m}{l} * a_y * \frac{l_{FA}}{c_{RA}} - \frac{l_{RA} * \dot{\psi}}{v}$$

(15) equalize

$$\delta = \frac{m}{l} * a_y * \frac{l_{RA}}{c_{FA}} - \frac{m}{l} * a_y * \frac{l_{FA}}{c_{RA}} + \frac{l_{FA} * \dot{\psi}}{v} + \frac{l_{RA} * \dot{\psi}}{v}$$

Basic vehicle dynamics calculation and vehicle models

(16) $$\delta = \frac{m}{l} * a_y * \frac{l_{RA}}{c_{FA}} - \frac{m}{l} * a_y * \frac{l_{FA}}{c_{RA}} + \frac{l_{FA} * \dot{\psi}}{v} + \frac{l_{RA} * \dot{\psi}}{v}$$

(17) $$\delta = \frac{m}{l} * a_y * \left[\frac{l_{RA}}{c_{FA}} - \frac{l_{FA}}{c_{RA}}\right] + \frac{\dot{\psi}}{v} * (l_{FA} + l_{RA})$$

$$\frac{\dot{\psi}}{v} \rightarrow \frac{1}{r}, \qquad (l_{FA} + l_{RA}) \rightarrow l$$

$$\delta = \frac{l}{r} + \frac{m}{l} * a_y * \left[\frac{l_{RA}}{c_{FA}} - \frac{l_{FA}}{c_{RA}}\right]$$

Steady-state cornering

(18) $$\delta = \frac{l}{r} + \frac{m}{l} * a_y * \left[\frac{l_{RA}}{c_{FA}} - \frac{l_{FA}}{c_{RA}}\right]$$

Ackermann

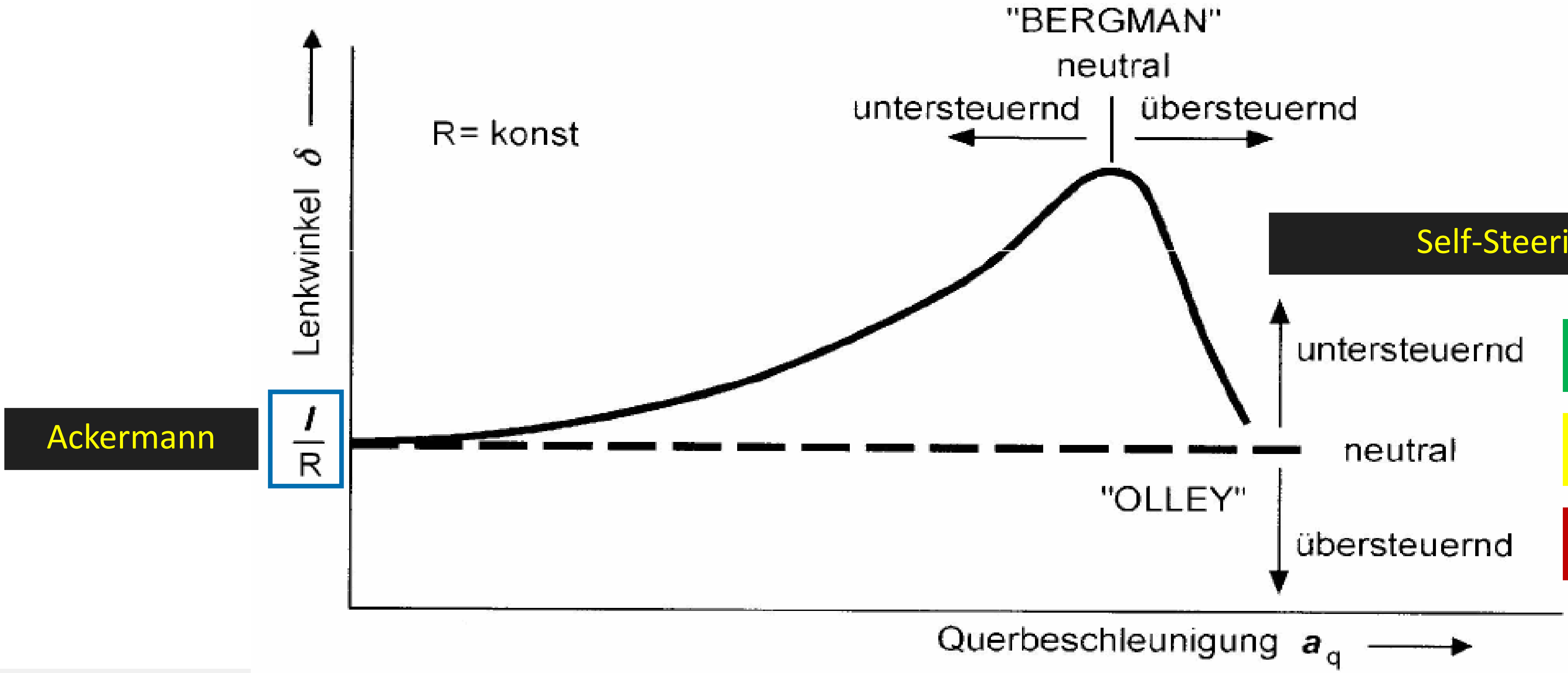
Self-Steering Behavior

(19) $$\delta = \frac{l}{r} + \Delta\alpha \qquad (\Delta\alpha = \alpha_{FA} - \alpha_{RA})$$

Understeer and oversteer definition

Ackermann | Self-Steering Behavior

$$\delta = \frac{l}{r} + \Delta\alpha \quad (\Delta\alpha = \alpha_{FA} - \alpha_{RA})$$



$\alpha_{FA} - \alpha_{RA} > 0$

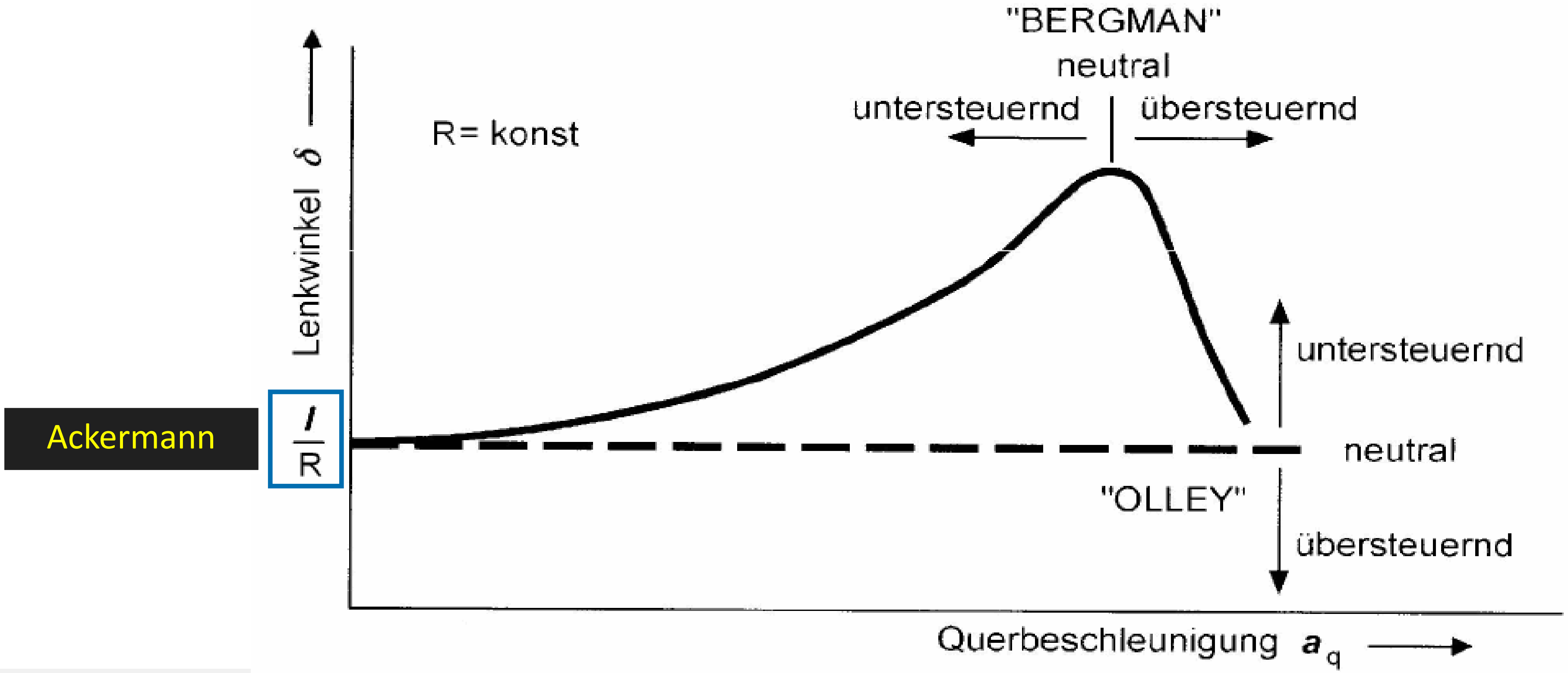
$\alpha_{FA} - \alpha_{RA} = 0$

$\alpha_{FA} - \alpha_{RA} < 0$

Understeer and oversteer definition

Ackermann | Self-Steering Behavior

$$\delta = \frac{l}{r} + \Delta\alpha \quad (\Delta\alpha = \alpha_{FA} - \alpha_{RA})$$



Definition of Eigenlenkgradient (Understeer Gradient)

Calculation of Eigenlenkgradient

(20)

$$EG = \frac{d\delta}{da_y} - \frac{d\delta_A}{da_y} = \frac{1}{i_s} * \frac{d\delta_H}{da_y} - \frac{d\delta_A}{da_y}$$

$= 0$ if $R = const$ since $d\delta_A \approx \frac{l}{R}$

Calculation of **the EG** = Eigenlenkgradient (engl. understeer gradient)
$\boldsymbol{d\delta_H}$: Steering wheel angle
$\boldsymbol{d\delta_A}$: Ackermann angle

Calculation of specific Eigenlenkgradient

(21)

$$EG_H = \frac{d\delta_H}{da_y} - \frac{d\delta_A}{da_y}$$

H = Hand at Steering Wheel

Definition of Eigenlenkgradient (Understeer Gradient)

Calculation of Eigenlenkgradient

(20)
$$EG = \frac{d\delta}{da_y} - \frac{d\delta_A}{da_y} = \frac{1}{i_s} * \frac{d\delta_H}{da_y} - \frac{d\delta_A}{da_y}$$

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Calculation of **the EG** = Eigenlenkgradient (engl. understeer gradient)
$\boldsymbol{d\delta_H}$: Steering wheel angle
$\boldsymbol{d\delta_A}$: Ackermann angle

Calculation of specific Eigenlenkgradient

(21)
$$EG_H = \frac{d\delta_H}{da_y} - \frac{d\delta_A}{da_y}$$

H = Hand at Steering Wheel

$EG = 0$: Neutral
$EG < 0$: Oversteer
$EG > 0$: Understeer

Characteristic Velocity and Critical Velocity

Calculation of Characteristic Velocity

(22)

$$\left[\frac{\dot{\psi}}{\delta_H}\right]_{v_{char}} = \frac{1}{2} * \left[\frac{\dot{\psi}}{\delta_H}\right]_{EG=0}$$

if $\frac{\dot{\psi}}{\delta_H} > \left[\frac{\dot{\psi}}{\delta_H}\right]_{EG=0}$ and $R = const$

i.e. if vehicle shows steady-state understeer

Calculation of Critical Velocity

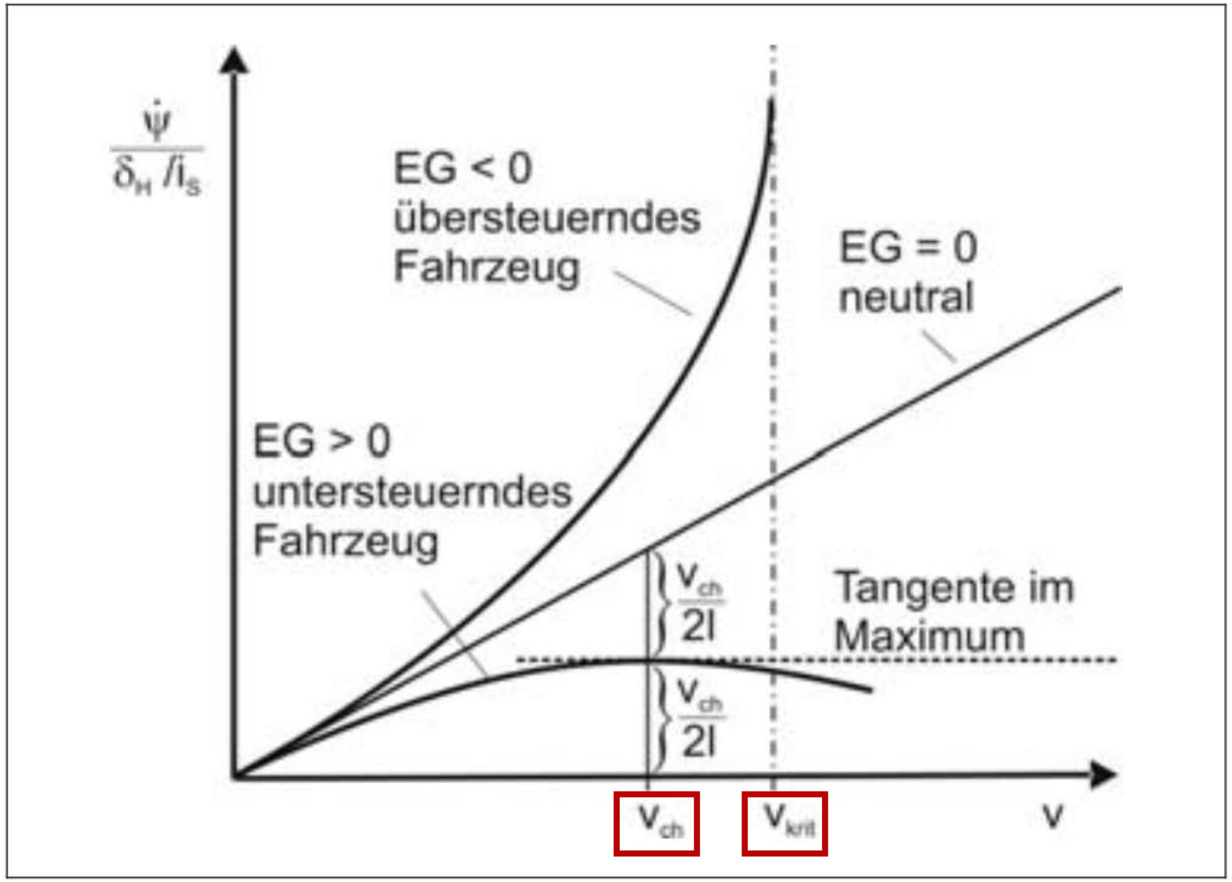
(23)

$$\left[\frac{\dot{\psi}}{\delta_H}\right]_{v_{crit}} = \frac{1}{2} * \left[\frac{\dot{\psi}}{\delta_H}\right]_{EG=0}$$

if $\frac{\dot{\psi}}{\delta_H} < \left[\frac{\dot{\psi}}{\delta_H}\right]_{EG=0}$ and $R = const$

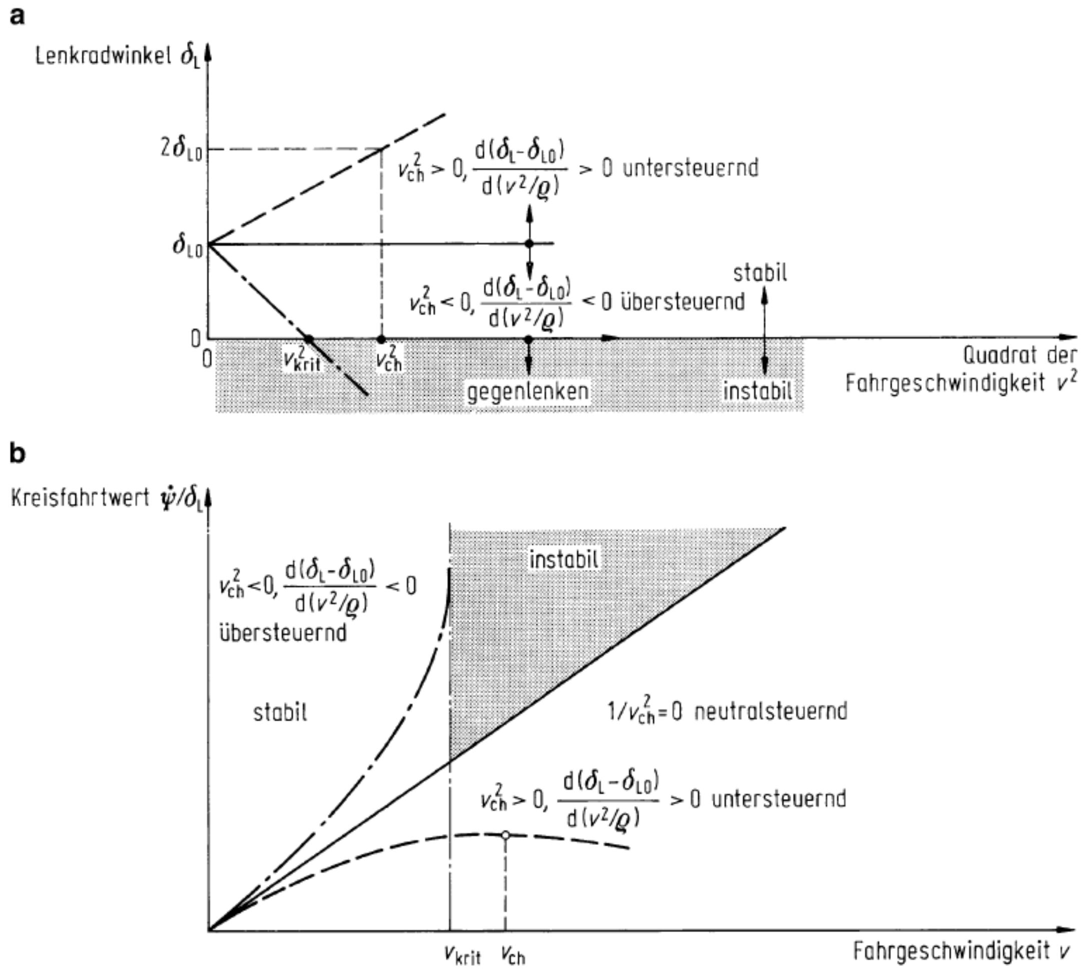
i.e. if vehicle shows steady-state oversteer

Characteristic Velocity and Critical Velocity



[4]

Characteristic Velocity and Critical Velocity



[5]

Practice Session: 15 min

- You have a vehicle with the following data:
  - Mass m = 1600 kg
  - Wheelbase = 2540 mm
  - Track width = 1420 mm
  - m_{FA} = 880 kg
  - Steering ratio = 1/15
  - Yaw inertia J_z = 2800 kgm²
  - Cornering stiffness = 3000 N/°

- Calculate the following:
  1. CoG in x-direction
  2. Ackermann steering angle for constant cornering at R = 100 m
  3. EG for 2. between 0 and 4 m/s²

Practice Session: 15 min

- How much steering wheel angle is necessary if your driver wants to corner at a steady acceleration of 4 m/s^2?
- Which three measures do you recommend to tune the vehicle towards less understeer (assuming only knowledge of the single track model)?
- How would you achieve an increase in understeering gradient by 50%?

Practice Session – Answers

1. $l_{RA} = \frac{m_{FA} * l}{m} = \frac{880\ kg * 2{,}54\ m}{1600\ kg}$

2. $\delta_A = \frac{l}{R} = \frac{2{,}54\ m}{100\ m} = 0{,}0254\ rad = 1{,}455°$

$$\delta_{A,H} = \frac{\delta}{i_s} = 15 * 1{,}455° = 21{,}825°$$

3. $EG = \frac{1}{i_s} * \frac{d\delta_H}{da_y} - \frac{d\delta_A}{da_y}$ — 0, da R = const

$$d\delta_H = \delta_{H,4m/s^2} - \delta_{H,0m/s^2} = \delta_{H,4m/s^2} - \delta_A$$

→ Nur Eigenlenkwinkelbedarf für EG

$$da_y = 4\frac{m}{s^2} - 0\frac{m}{s^2}$$

Practice Session – Answers

3. $EG_{0-4m/s^2} = \frac{\left(\delta_A + \delta_{Eigenlenk,\frac{4m}{s^2}}\right) - \delta_A}{4\frac{m}{s^2}}$

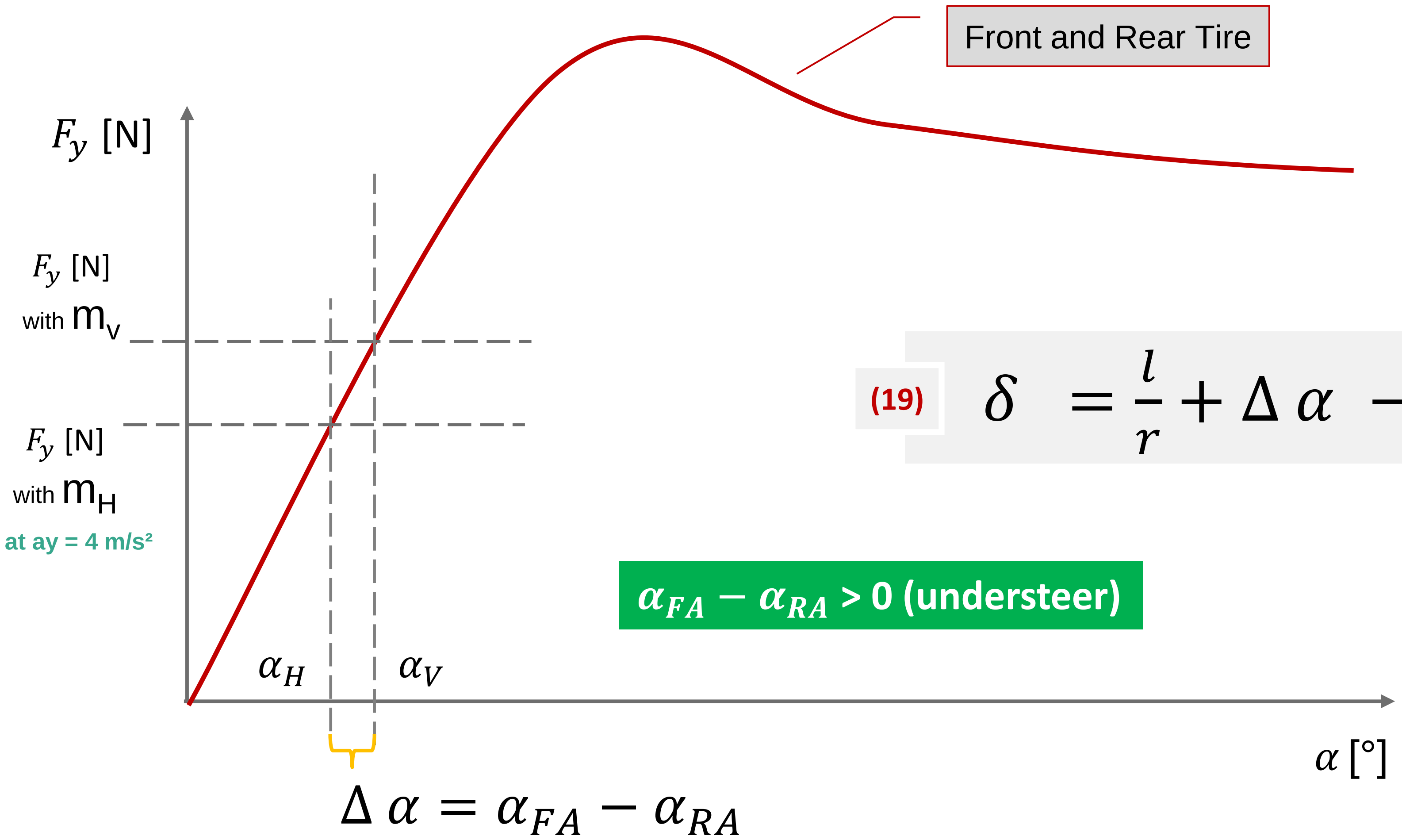
$$EG = \frac{1}{4\,\frac{m}{s^2}} * \left[\frac{2{,}54\ \mathrm{m}}{100\ \mathrm{m}} + \frac{1600\ \mathrm{kg}}{2{,}54\ \mathrm{m}} * 4\,\frac{\mathrm{m}}{\mathrm{s}^2} * \left[\frac{1{,}397\ m}{3000\ N/°} - \frac{1{,}143\ m}{3000\ N/°}\right] - \frac{2{,}54\ \mathrm{m}}{100\ \mathrm{m}}\right]$$

Ackermannlenkwinkel

Eigenlenkwinkelbedarf

→ $EG = \frac{1}{4\frac{m}{s^2}} * [1{,}455° + 0{,}213° - 1{,}455°] = 0{,}053\ \frac{°}{\frac{m}{s^2}}$

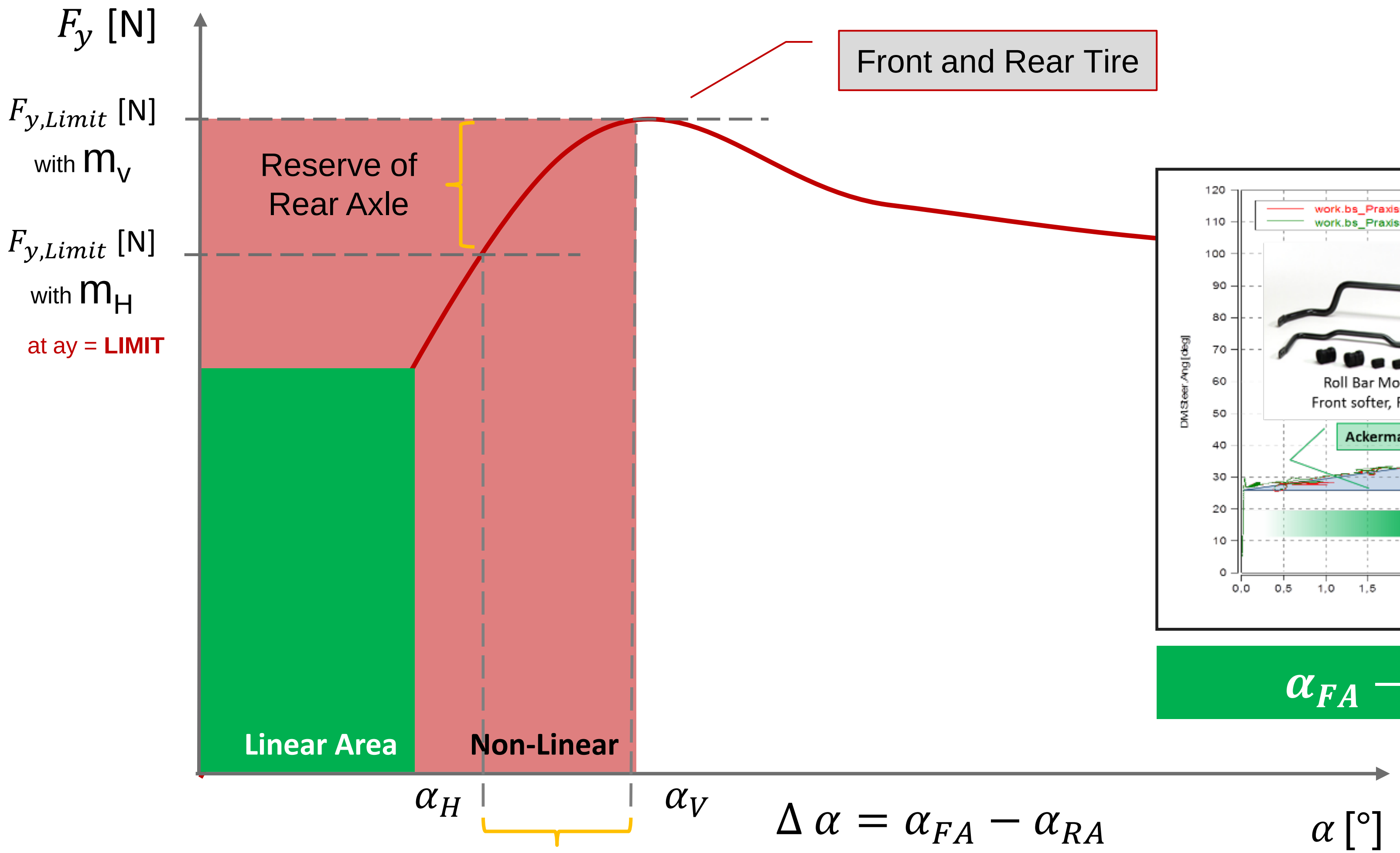
Case 1: Understeer behavior with wheel load distribution

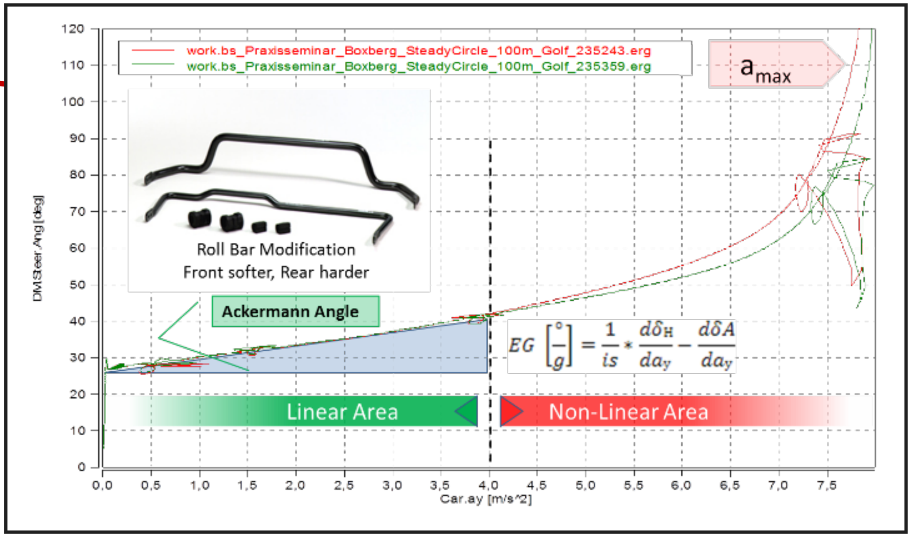


(19) $$\delta \;=\frac{l}{r}+\Delta\,\alpha \;\rightarrow [\alpha_{FA}-\alpha_{RA}]$$

$\alpha_{FA}-\alpha_{RA}$ > 0 (understeer)

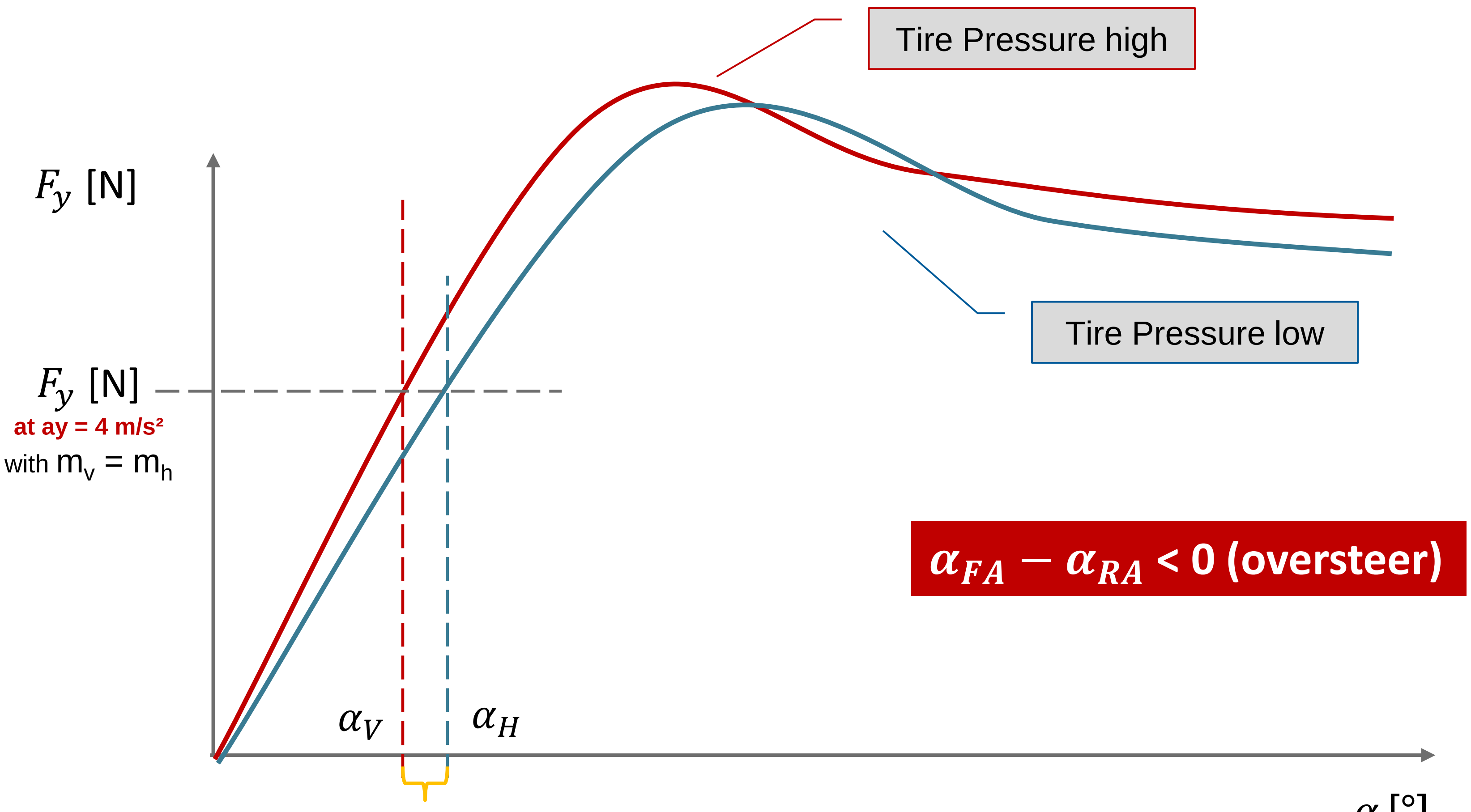
Case 1: Understeer behavior with wheel load distribution





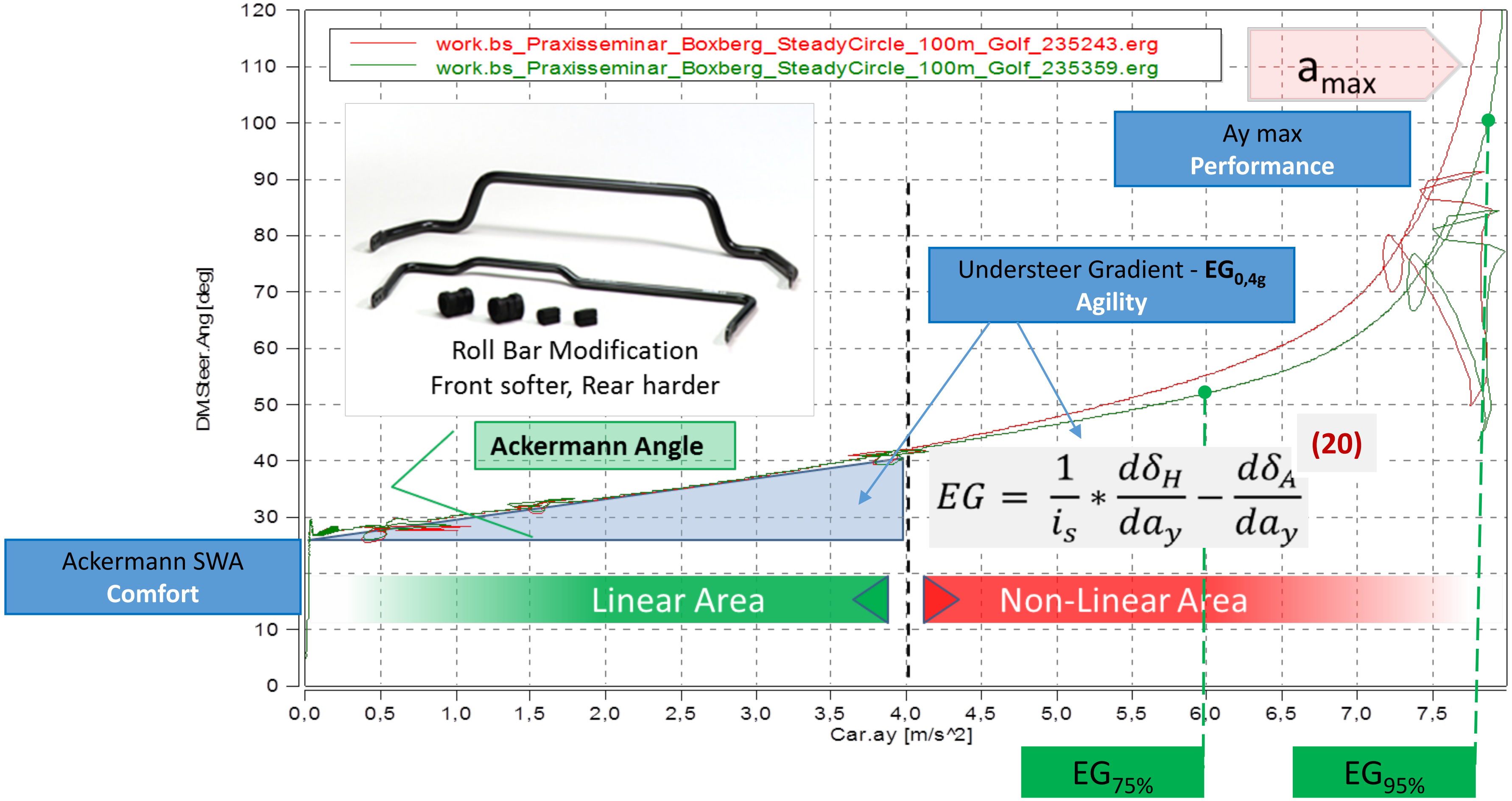
$\alpha_{FA} - \alpha_{RA}$ > 0 (understeer)

Case 2: Oversteer behavior with tire pressure difference

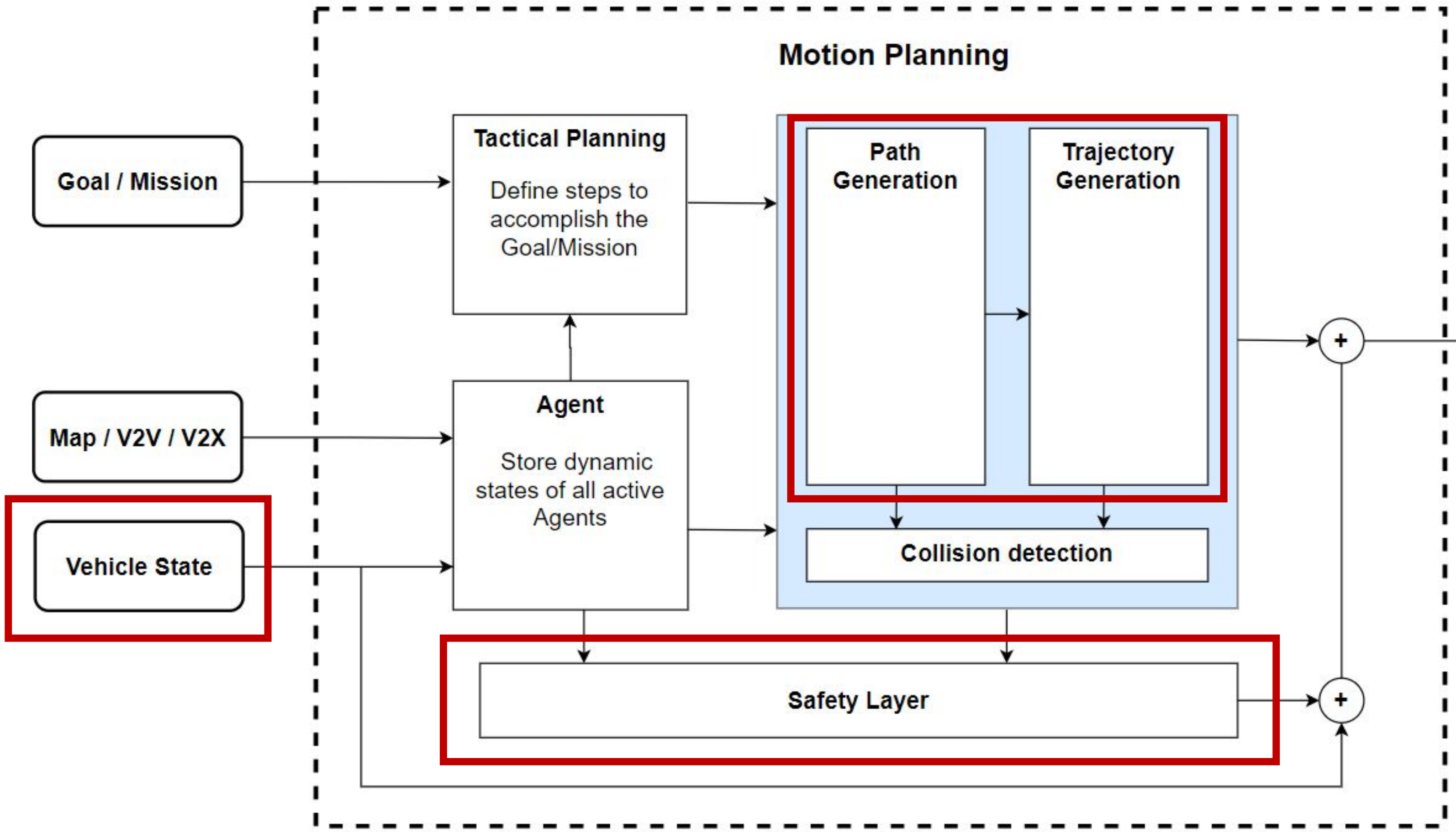


$$\Delta \alpha = \alpha_{FA} - \alpha_{RA}$$

Model behavior in steady-state cornering

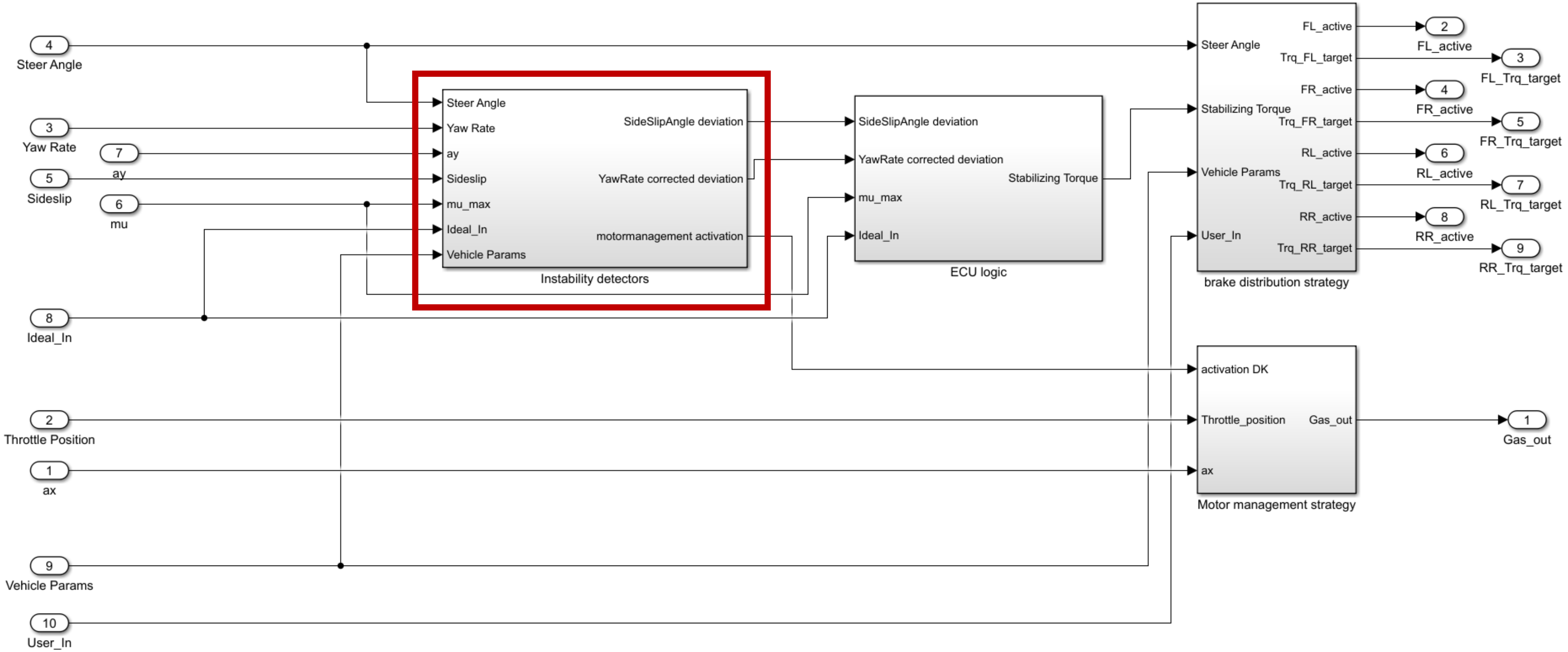


Use-Cases for a Single-track model – Model-based motion planning

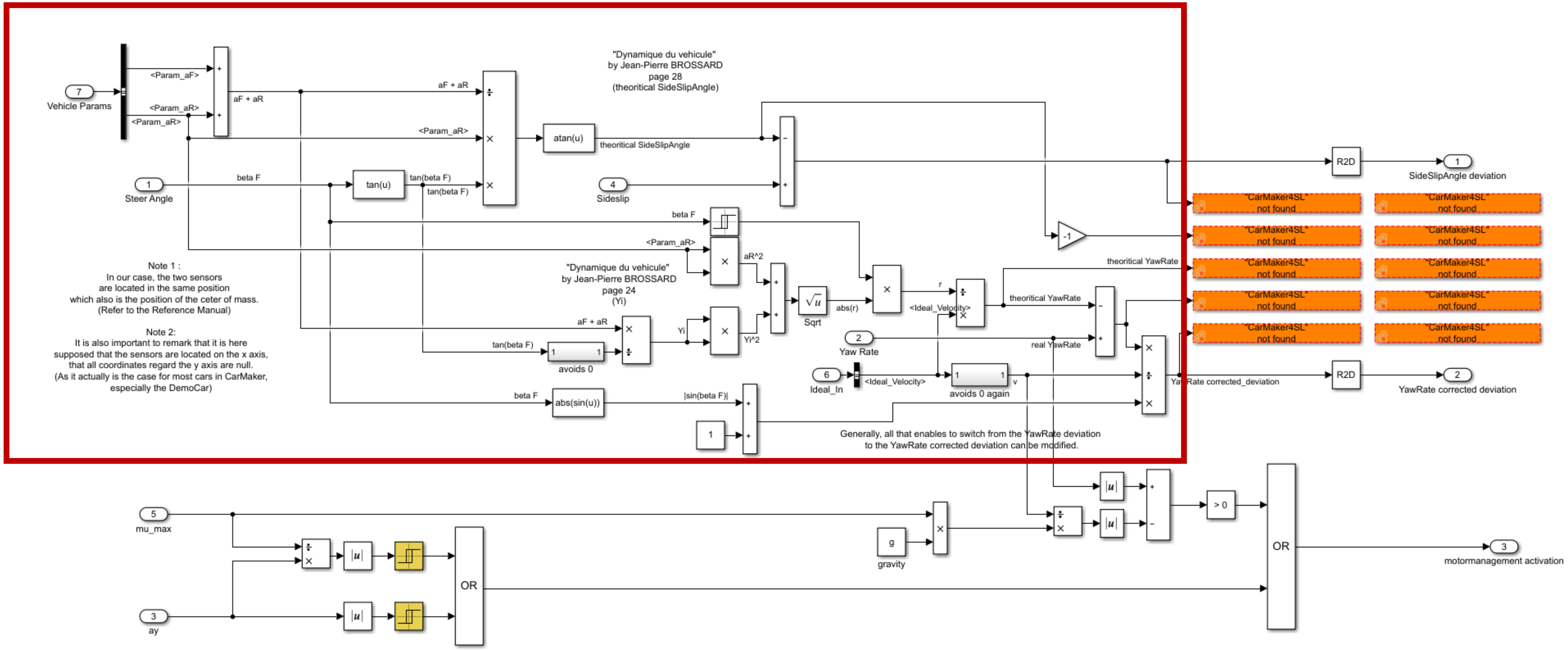


Problem:
Computational power/cost limits model complexity

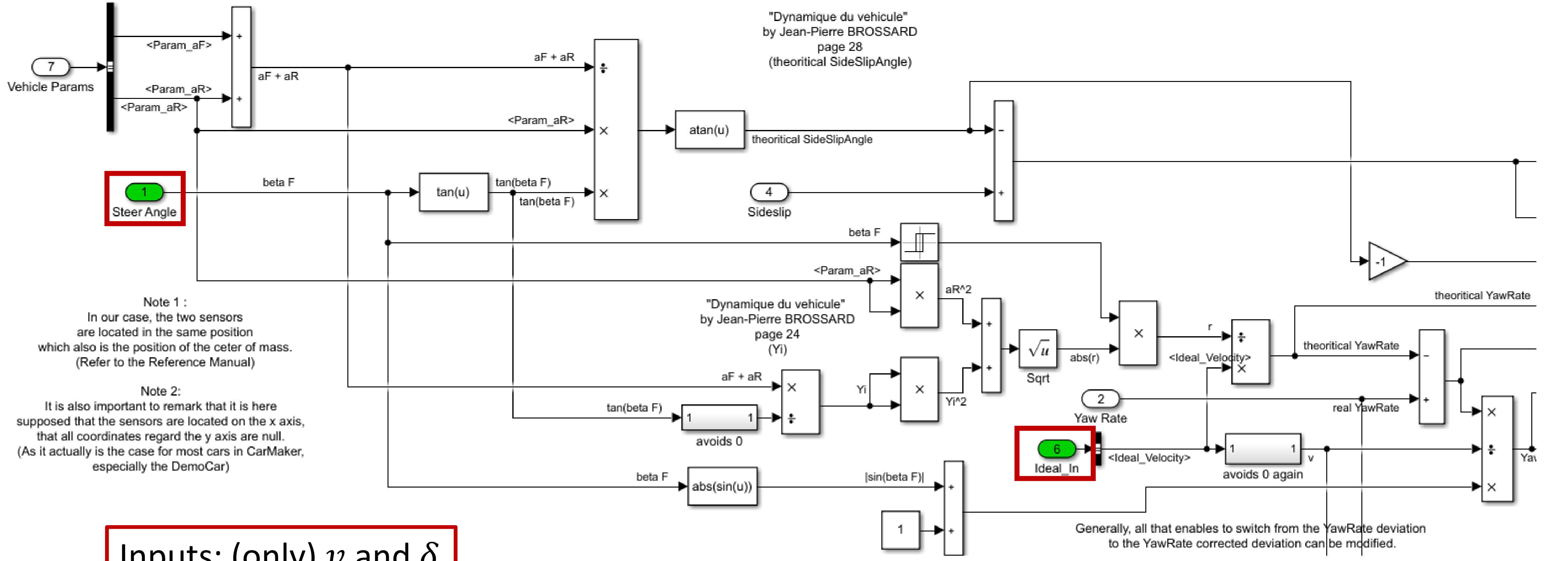
Use-Cases for a Single-track model – Model-based control



Use-Cases for a Single-track model – Model-based control

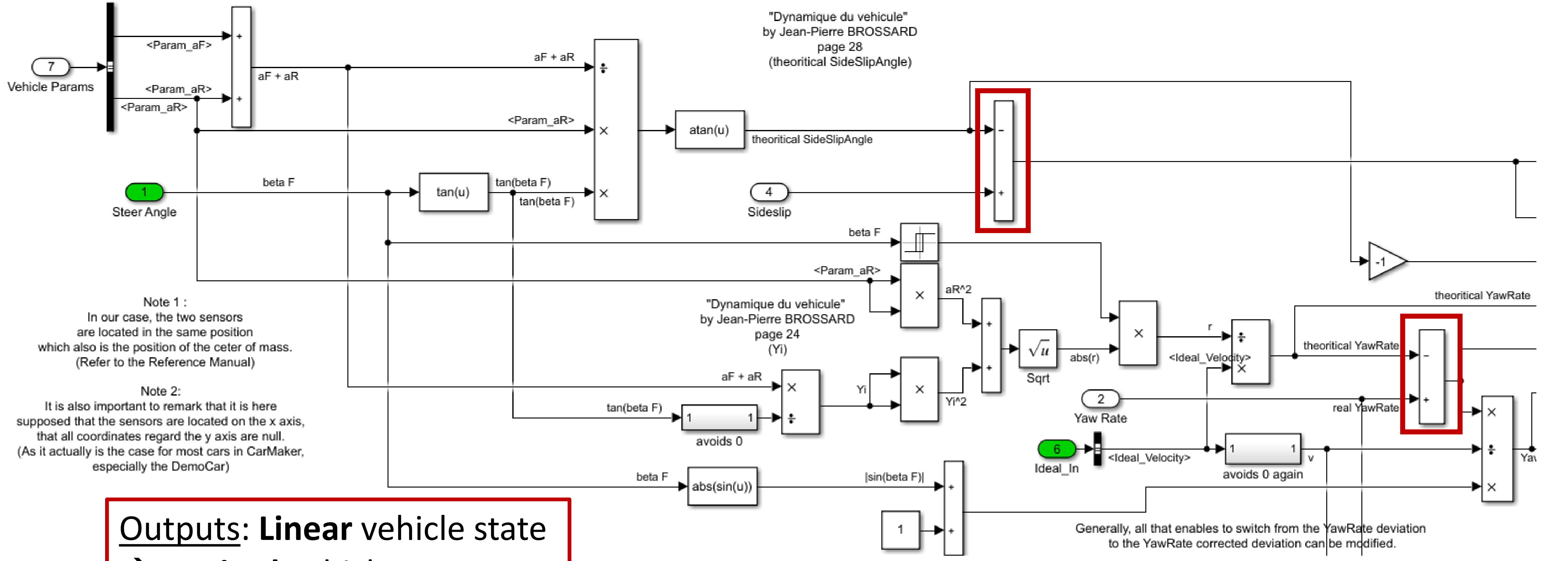


Use-Cases for a Single-track model – Model-based control



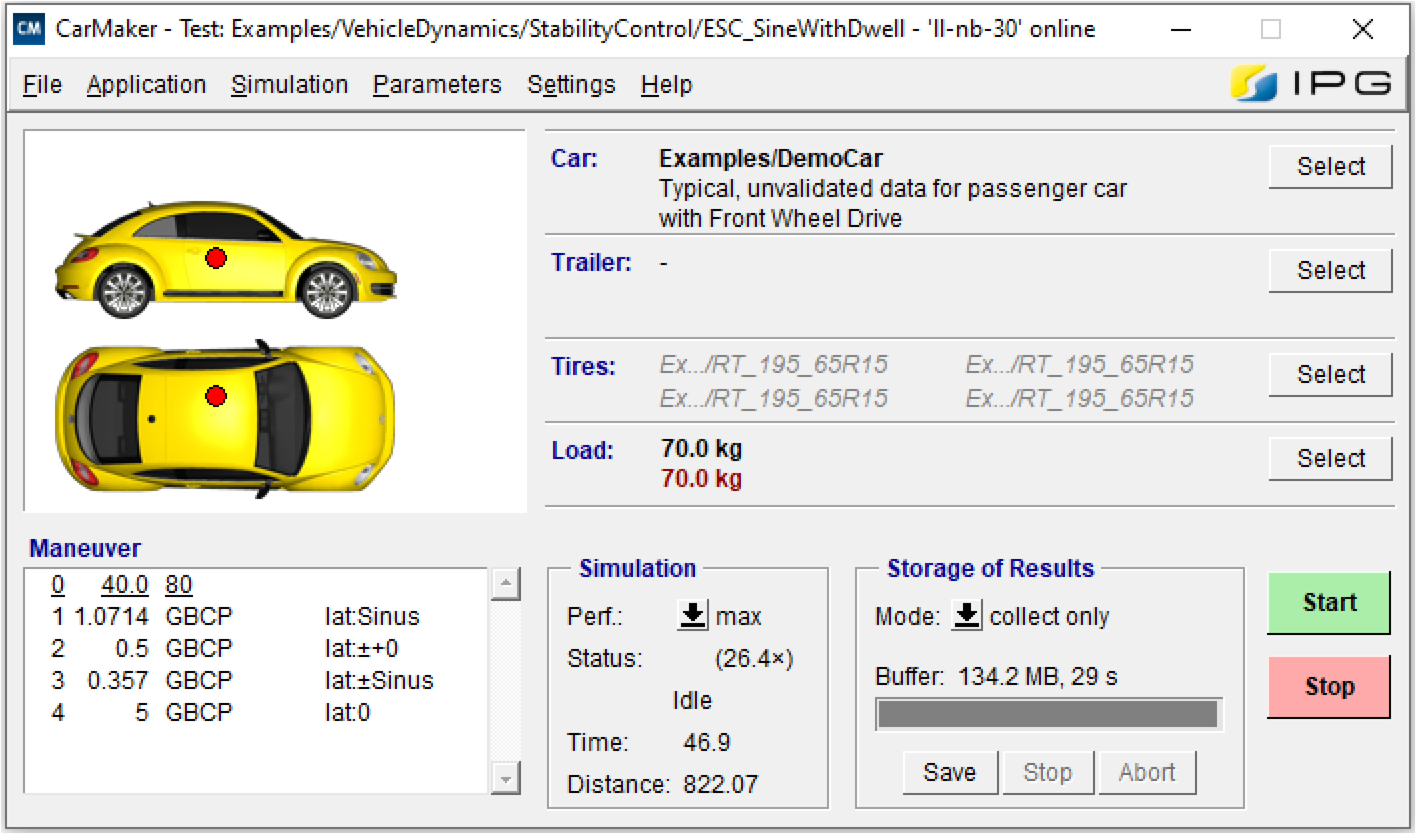
<u>Inputs</u>: (only) v and δ

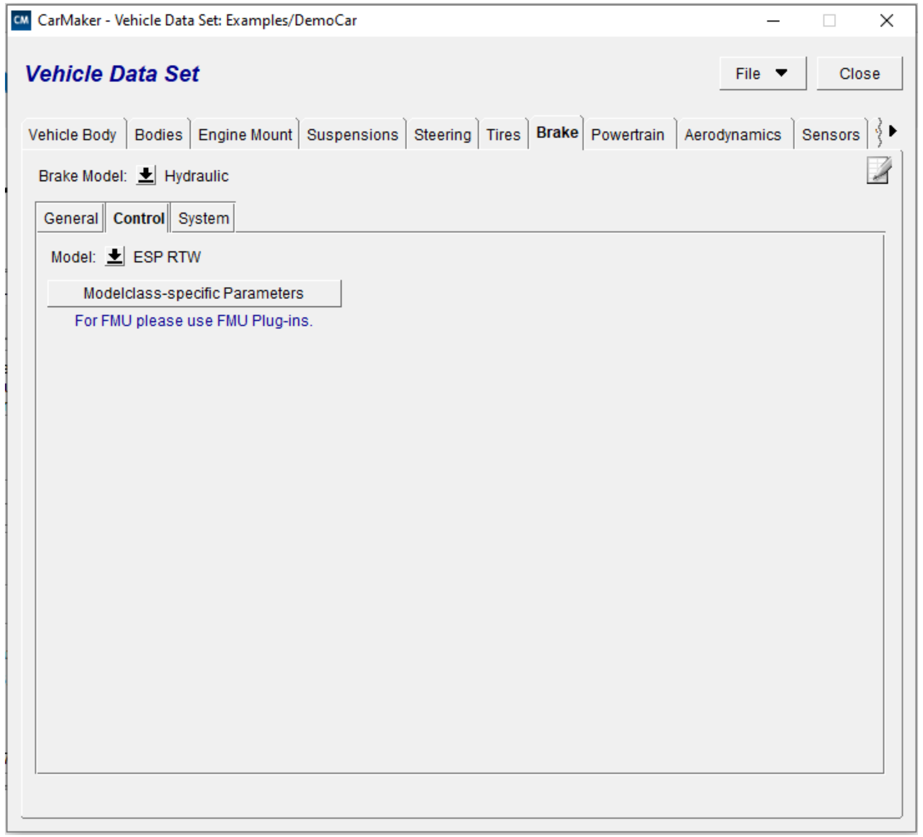
Use-Cases for a Single-track model – Model-based control



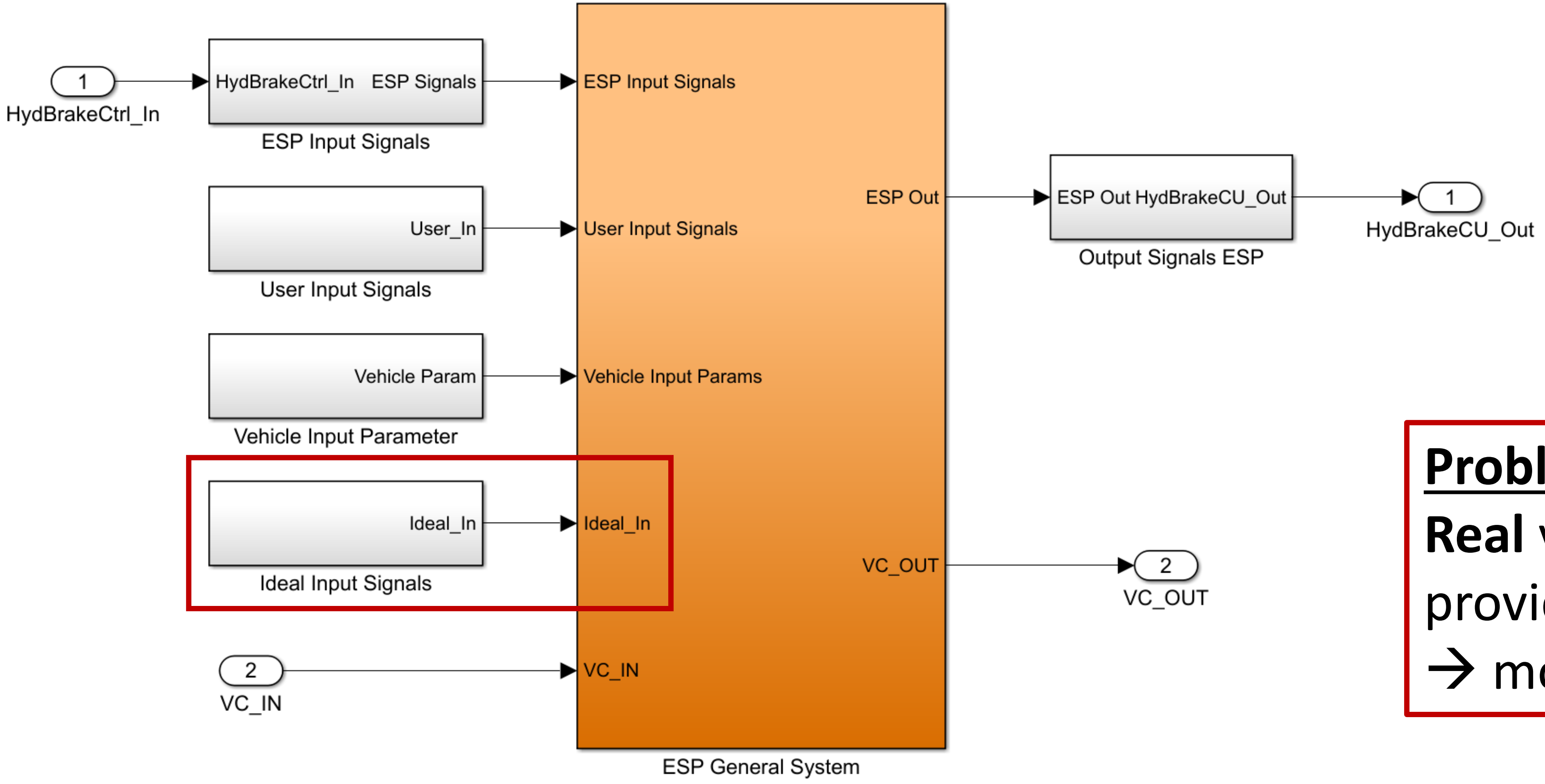
Outputs: **Linear** vehicle state
→ **Desired** vehicle response

Model-based control – CarMaker example



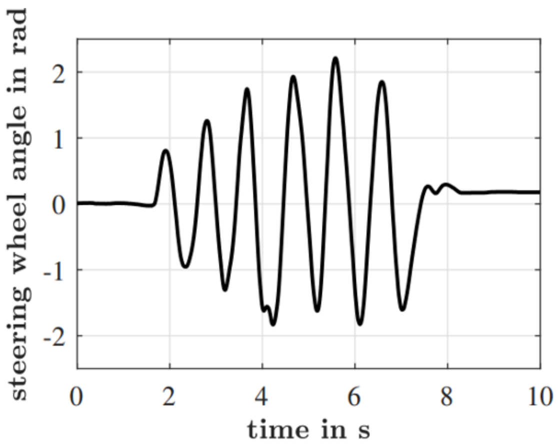


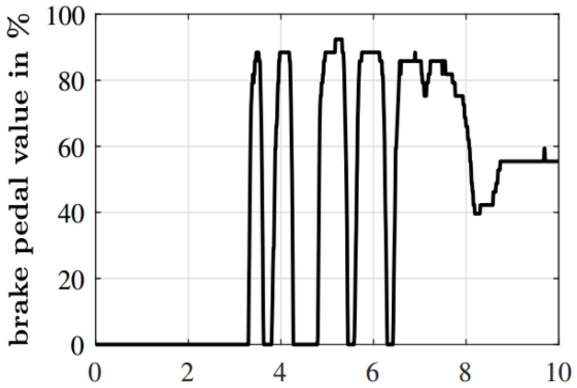
Use-Cases for a Single-track model – Vehicle state estimation



Problem:
Real vehicles don't provide **ideal** in-/outputs
→ model-based estimate

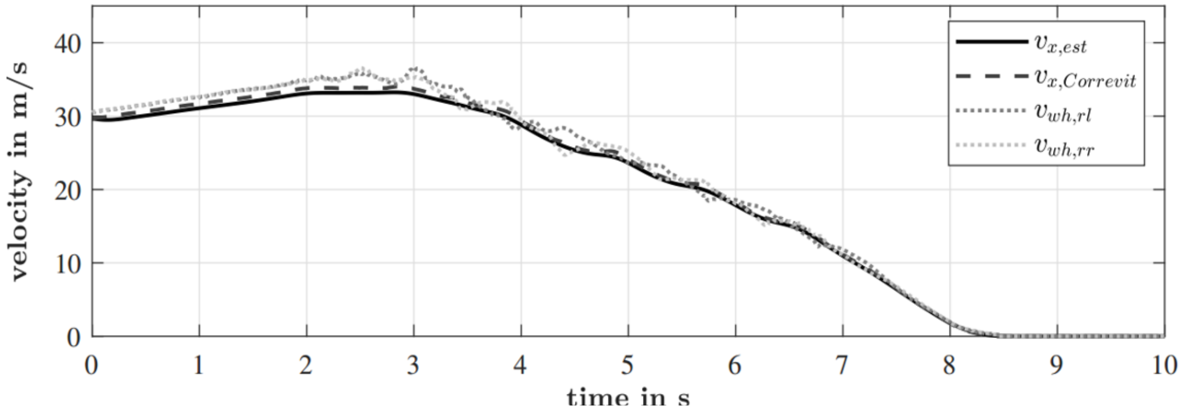
Use-Cases for a Single-track model – Vehicle state estimation





Estimation of non-measurable quantities

- **Longitudinal velocity** under slip
- **Lateral velocity**
- Tire **forces**
- Available **friction coefficient** (tire potential)
- Road bank angle
- …



Prediction of vehicle state

- Motion planning
- Advanced control

What can we improve? – Extensions of the Single-Track model

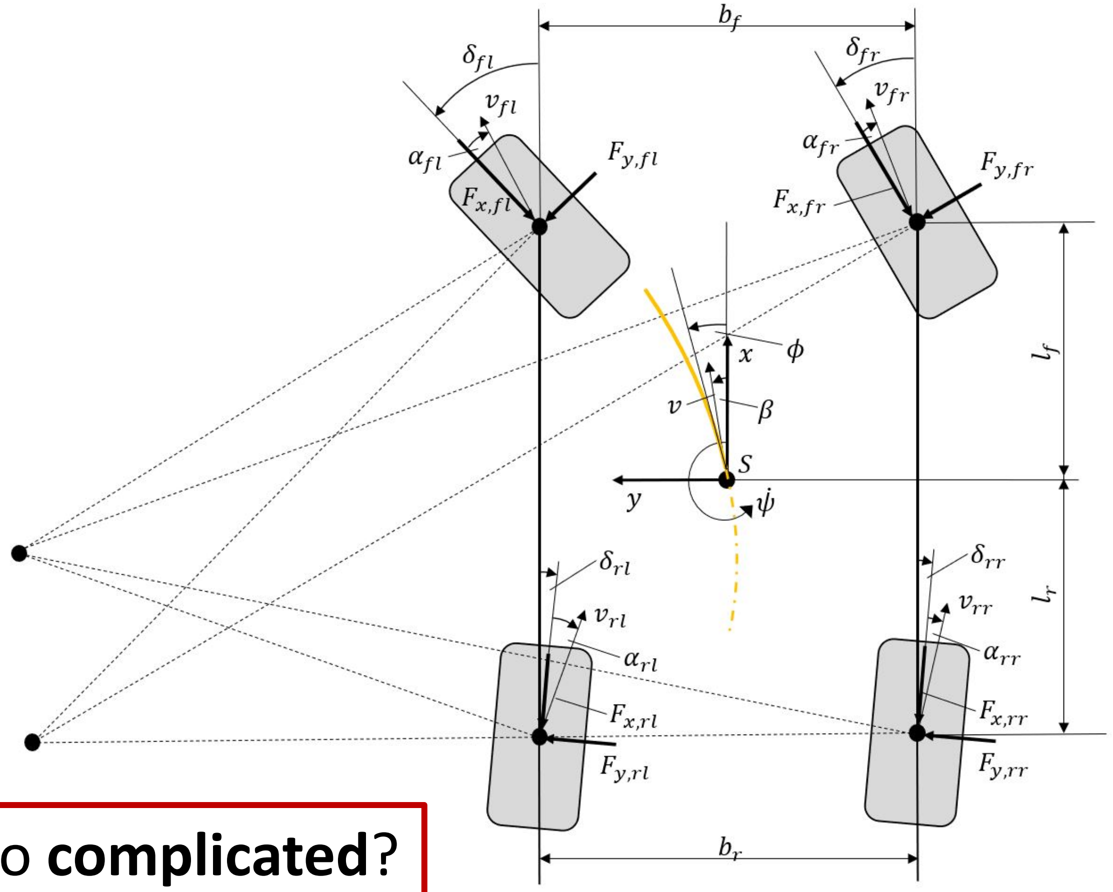
<u>Two-track model (**6 body-DOF**)</u>

- Consider **roll, heave** and **pitch** motion
  → **CoG height** is now relevant
  → Four wheels with **dynamic wheel load transfer**

<u>Non-linear tire modeling</u>

- Saturation through long./lat. force limits (tire potential)

- Steering angle on both axles
- Slip-angle contribution of resulting axle stiffness
- Transient vertical dynamics
- Combined slip modeling
- …



→ How **complex** is too **complicated**?

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