Lernziel:

Sie erinnern sich an die Ableitungsregeln und können diese anwenden.

1. Leiten Sie ab. Verwenden Sie die Faktor- und die Summenregel.

a) $f(x) = 3x^2 + 1$ → $\dot{f}(x) = 6x$

b) $f(x) = 11x^3 + x$+g → $\dot{f}(x) = 33x^2 + 1$

c) $f(x) = 4x^5 + 8x^4 + x^3 + \frac{3}{4}x - 222y$ → $\dot{f}(x) = 20x^4 + 32x^3 + 3x^2 + \frac{3}{4}$

d) $f(r) = \frac{1}{2}xr^7 + yr^5 - ar^2$ → $\dot{f}(r) = \frac{7}{2}xr^6 + 5yr^4 - 2ar$

e) $f(a) = a^b - a^{7b} + 90a + 231$ → $\dot{f}(a) = ba^{b-1} - 7ba^{7b-1} + 90$

f) $f(t) = \frac{4}{3}t^{10} + 2t^{77} - 3t^5 + t^0$ → $\dot{f}(t) = \frac{40}{3}t^9 + 154t^{76} - 15t^4$

2. Leiten Sie ab. Verwenden Sie die Produktregel.

a) $f(x) = (2x+3) * (x-1)$

→ $\dot{f}(x) = 2(x-1) + (2x+3) * 1 = 2x - 2 + 2x + 3 = \mathbf{4x + 1}$

b) $f(x) = (x^2+5) * (x+8)$

→ $\dot{f}(x) = 2x * (x+8) + (x^2+5) * 1 = 2x^2 + 16x + x^2 + 5 = \mathbf{3x^2 + 16x + 5}$

c) $f(x) = (3x^3 + 9x) * (x^2 - 1)$

→ $\dot{f}(x) = (9x^2 + 9)(x^2 - 1) + (3x^3 + 9x) * 2x = 9x^4 + 9x^2 - 9x^2 - 9 + 6x^4 + 18x^2 =$

$= \mathbf{15x^4 + 18x^2 - 9}$

d) $f(x) = (x^3 + x^2 + 1) * (x^3 + 99)$

→ $\dot{f}(x) = (3x^2 + 2x)(x^3 + 99) + (x^3 + x^2 + 1) * 3x^2 =$

$= 3x^5 + 297x^2 + 2x^4 + 198x + 3x^5 + 3x^4 + 3x^2 =$

$= \mathbf{6x^5 + 5x^4 + 300x^2 + 198x}$

e) $f(x) = x^2(30x^3 + 2x^2 + x + 7)$

→ $\dot{f}(x) = 2x(30x^3 + 2x^2 + x + 7) + x^2(90x^2 + 4x + 1)$

$= 60x^4 + 4x^3 + 2x^2 + 14x + 90x^4 + 4x^3 + x^2 =$

$= \mathbf{150x^4 + 8x^3 + 3x^2 + 14x}$

f) $f(x) = (x^4 + x^2) * (x^{66} - 1)$

→ $\dot{f}(x) = (4x^3 + 2x) * (x^{66} - 1) + (x^4 + x^2) * 66x^{65} =$

$= 4x^{69} - 4x^3 + 2x^{67} - 2x + 66x^{69} + 66x^{67} =$

$= \mathbf{70x^{69} + 68x^{67} - 4x^3 - 2x}$

3. Leiten Sie ab. Verwenden Sie die Quotientenregel.

a) $f(x) = \frac{x^2 - 2x + 1}{x - 1}$

→ $\dot{f}(x) = \frac{(2x-2)(x-1) - (x^2 - 2x + 1) * 1}{(x-1)^2} = \frac{(2x-2) - (x-1) * 1}{(x-1)} = \frac{2 * (x-1) - (x-1) * 1}{(x-1)} = 2 - 1 = \mathbf{1}$

b) $f(x) = \frac{x^2-2x+1}{x+1}$

→ $\dot{f}(x) = \frac{(2x-2)(x+1)-(x^2-2x+1)*1}{(x+1)^2} = \frac{2x^2+2x-2x-2-x^2+2x-1}{(x+1)^2} = \boldsymbol{\frac{x^2+2x-3}{(x+1)^2}}$

c) $f(x) = \frac{x^2+1}{x+1}$

→ $\dot{f}(x) = \frac{2x(x+1)-(x^2+1)*1}{(x+1)^2} = \frac{2x^2+2x-x^2-1}{(x+1)^2} = \boldsymbol{\frac{x^2+2x-1}{(x+1)^2}}$

d) $f(x) = \frac{4x^2+12x+}{3x+3}$

→ $\dot{f}(x) = \frac{(8x+12)(3x+3)-(4x^2+12x+\ )*3}{(3x+3)^2} = \frac{24x^3+24x+36x+36- \quad {}^3-36x}{(3x+3)^2} = \boldsymbol{\frac{12x^2+36x+9}{(3x+3)^2}}$

4. Leiten Sie ab. Verwenden Sie die Kettenregel.

a) $f(x) = (x+1)^3$

→ $\dot{f}(x) = \boldsymbol{3(x+1)^2}$

b) $f(x) = \frac{1}{2}(x^2-2)^2$

→ $\dot{f}(x) = (x^2-2) * 2x = \boldsymbol{2x^3 - 4x^2}$

c) $f(x) = (x^2+x+77)^2$

→ $\dot{f}(x) = 2(x^2+x+77) * (2x+1) = 4x^3 + 2x^2 + 4x^2 + 2x + 288x + 144$

$= \boldsymbol{4x^3 + 6x^2 + 290x + 144}$

d) $f(a) = (a^3+a^2+13)^7$

→ $\dot{f}(x) = \boldsymbol{7(a^3+a^2+13)^6 * (3a^2+2a)}$

5. Leiten Sie ab.

a) $f(x) = \frac{(x+1)^2}{(x-1)^2} = \left(\frac{x+1}{x-1}\right)^2$

→ $\dot{f}(x) = 2 * \left(\frac{x+1}{x-1}\right)\frac{x-1-x-1}{(x-1)^2} = 2 * \left(\frac{x+1}{x-1}\right)\frac{-2}{(x-1)^2} = \boldsymbol{\frac{-4(x+1)}{(x-1)^3}}$

b) $f(x) = \sqrt{\frac{3+x}{3-x}} = \left(\frac{3+x}{3-x}\right)^{\frac{1}{2}}$

→ $\dot{f}(x) = \frac{1}{2} * \left(\frac{3+x}{3-x}\right)^{-\frac{1}{2}}\frac{(3-x)-(3+x)*(-1)}{(3-x)^2} = \frac{1}{2} * \left(\frac{3+x}{3-x}\right)^{-\frac{1}{2}}\frac{6}{(3-x)^2}$

$= \frac{1}{2} * \sqrt{\frac{3-x}{3+x}}\frac{6}{(3-x)^2} = \boldsymbol{\sqrt{\frac{3-x}{3+x}}\frac{3}{(3-x)^2}}$

c) $f(x) = \frac{(x^2+4x+4)^2}{x+2} = \frac{[(x+2)^2]^2}{x+2} = \frac{(x+2)^4}{x+2} = (x+2)^3$

$\boldsymbol{\dot{f}(x) = 3 * (x+2)^2}$

d) $f(x) = \left[\left(x^3 - 7x^2 + \sqrt{x}\right)^7\right]^2$

$\rightarrow \dot{f}(x) = \mathbf{14} * \left(\boldsymbol{x^3 - 7x^2 + \sqrt{x}}\right)^{\mathbf{13}} * (\boldsymbol{3x^2 - 14x + \frac{1}{2\sqrt{x}}})$

e) $f(x) = \frac{\sqrt{x^2+1}}{x^3}$

$\rightarrow \dot{f}(x) = \frac{\frac{1}{2}(x^2+1)^{-\frac{1}{2}}*2x*x^3-\sqrt{x^2+1}*3x^2}{(x^3)^2} = \frac{(x^2+1)^{-\frac{1}{2}}x*x-\sqrt{x^2+1}*3}{x^4} = \frac{(x^2+1)^{-\frac{1}{2}}*x^2-\sqrt{x^2+1}*3}{x^4}$

$= \frac{\frac{x^2}{\sqrt{x^2+1}} - \sqrt{x^2+1} * 3}{x^4} = \frac{\frac{x^2-3x^2-3}{\sqrt{x^2+1}}}{x^4} = \frac{\boldsymbol{-2x^2 - 3}}{\boldsymbol{x^4\sqrt{x^2+1}}}$

f) $f(x) = \frac{(9x^2+7)*(x^3+2)}{\sqrt{x+1}} = \frac{9x^5+18x^2+7x^3+14}{\sqrt{x+1}}$

$\rightarrow \dot{f}(x) = \frac{(45x^4+36x+21x^2)*(x+1)^{\frac{1}{2}}-(9x^5+18x^2+7x^3+14)*\frac{1}{2}(x+1)^{-\frac{1}{2}}}{x+1} =$

$= \frac{45x^4 + 21x^2 + 36x}{\sqrt{x+1}} - \frac{9x^5 + 7x^3 + 18x^2 + 14}{2(x+1)^{\frac{3}{2}}}$

$= \frac{\boldsymbol{81x^5 + 90x^4 + 35x^3 + 96x^2 + 72x - 14}}{\boldsymbol{2(x+1)^{\frac{3}{2}}}}$

g) $f(x) = \frac{(x+1)^3}{x}$

$\rightarrow \dot{f}(x) = \frac{3(x+1)^2*x-(x+1)^3}{x^2} = \frac{3x(x^2+2x+1)-(x^3+x^2+2x^2+2x+x+1)}{x^2} =$

$= \frac{(3x^3 + 6x^2 + 3x) - (x^3 + 3x^2 + 3x + 1)}{x^2} = \frac{\boldsymbol{2x^3 + 3x^2 - 1}}{\boldsymbol{x^2}}$

h) $f(x) = \sqrt{x - d^3 + 4}^{33} = \left[(x - d^3 + 4)^{\frac{1}{2}}\right]^{33} = (x - d^3 + 4)^{\frac{33}{2}}$

$\rightarrow \dot{f}(x) = \frac{\mathbf{33}}{\mathbf{2}}\left(\boldsymbol{x - d^3 + 4}\right)^{\frac{\mathbf{31}}{\mathbf{2}}}$

i) $f(x) = \frac{x^2+2x+1}{x+1}$

$\rightarrow f(x) = \frac{(x+1)^2}{(x+1)^2} = \mathbf{1}$

$\rightarrow \dot{f}(x) = \mathbf{0}$

j) $f(x) = \frac{x^2-1}{x+1}$

$\rightarrow f(x) = \frac{x^2-1}{x+1} = \frac{(x-1)(x+1)}{x+1} = \boldsymbol{x - 1}$

$\rightarrow \dot{f}(x) = \mathbf{1}$