Aufgaben: Trigonometrie Übungsblatt 2

1.

a)

$$\sin(60^\circ + \alpha) - \sin(60^\circ - \alpha)$$

$$\Rightarrow \sin(x_1 + x_2) - \sin(x_1 - x_2) = \underline{2 \cdot \cos x_1 \cdot \sin x_2} =$$

$$= 2 \cdot \cos(60^\circ) \cdot \sin\alpha = 2 \cdot \frac{1}{2} \cdot \sin\alpha = \underline{\underline{\sin\alpha}}$$

b)

$$\cos(\alpha + \beta) - \cos(\alpha - \beta) =$$

mit Additionstheorem: $\cos(x_1 + x_2) - \cos(x_1 - x_2) = -2 \cdot \sin x_1 \cdot \sin x_2$

$x_1 = \alpha \quad x_2 = \beta$

$$\Rightarrow \cos(\alpha + \beta) - \cos(\alpha - \beta) = -2 \cdot \sin\alpha \cdot \sin\beta$$

c)

$$\frac{\sin(\alpha + \beta)}{\cos\alpha \cdot \cos\beta} =$$

mit Additionstheoreme:

$$\cos x_1 \cdot \cos x_2 = \frac{1}{2} \cdot \left[\cos(x_1 - x_2) + \cos(x_1 + x_2)\right]$$

$$\frac{\sin(\alpha + \beta)}{\frac{1}{2}\left[\cos(\alpha - \beta) + \cos(\alpha + \beta)\right]} =$$

mit Additionstheorem:

$$\sin(\alpha + \beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$$

$$= \frac{\sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta}{\frac{1}{2}\left[\cos\alpha \cdot \cos\beta + \cancel{\sin\alpha \cdot \sin\beta} + \cos\alpha \cdot \cos\beta - \cancel{\sin\alpha \cdot \sin\beta}\right]} =$$

$$= \frac{\sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta}{\frac{1}{2}\left[2 \cdot \cos\alpha \cdot \cos\beta\right]} = \frac{\sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta}{\cos\alpha \cdot \cos\beta} =$$

$$= \frac{\sin\alpha \cdot \cancel{\cos\beta}}{\cos\alpha \cdot \cancel{\cos\beta}} + \frac{\cancel{\cos\alpha} \cdot \sin\beta}{\cancel{\cos\alpha} \cdot \cos\beta} = \frac{\sin\alpha}{\cos\alpha} + \frac{\sin\beta}{\cos\beta} = \underline{\tan\alpha + \tan\beta}$$

d)

$$\sin^4(\alpha) - \cos^4(\alpha) = \frac{1}{8}\left[\cancel{\cos(4x)} - 4 \cdot \cos(2x) + \cancel{3}\right] - \frac{1}{8}\left[\cancel{\cos(4x)} + 4 \cdot \cos(2x) + \cancel{3}\right]$$

$$= -\frac{1}{8} \cdot 4 \cdot \cos(2x) - \frac{1}{8} \cdot 4 \cdot \cos(2x) = -\cos(2\alpha)$$

e)

$$\frac{1 \cdot \tan\alpha}{1 + \tan^2\alpha} =$$

mit Additionstheorem: $\cos\alpha = \sqrt{\frac{1}{\tan^2\alpha + 1}} \Rightarrow \cos^2\alpha = \frac{1}{1 + \tan^2\alpha}$

$$= \cos^2\alpha \cdot \tan\alpha \qquad \text{mit} \quad \tan\alpha = \frac{\sin\alpha}{\cos\alpha}$$

$$= \cos^{\cancel{2}}\alpha \cdot \frac{\sin\alpha}{\cancel{\cos\alpha}} = \cos\alpha \cdot \sin\alpha$$

mit Additionstheorem: $\sin 2\alpha = 2 \cdot \sin\alpha \cdot \cos\alpha$

$$= \underline{\underline{\frac{1}{2}\sin 2\alpha}};$$

f)

$$\frac{1}{\sin^2\alpha} + \frac{1}{\cos^2\alpha} = \frac{2}{1 - \cos(2x)} + \frac{2}{1 + \cos 2\alpha}$$

$$= \frac{2\,(1 + \cos(2\alpha))}{(1 - \cos(2\alpha))(1 + \cos(2\alpha))} + \frac{2\,(1 - \cos(2\alpha))}{(1 + \cos 2\alpha)(1 - \cos 2\alpha)} = \frac{4}{1 - \cos^2(2\alpha)} = \underline{\underline{\frac{4}{\sin^2(2\alpha)}}};$$

weil $\sin^2\alpha + \cos^2\alpha = 1$

g)

$$\frac{1-\cos(2\alpha)}{\sin(2\alpha)} = \frac{1-\cos(2\alpha)}{2\cdot\sin\alpha\cdot\cos\alpha} = \frac{1-(1-2\sin^2\alpha)}{2\cdot\sin\alpha\cdot\cos\alpha} = \frac{\cancel{2}\cdot\sin^{\cancel{2}}\alpha}{\cancel{2}\cdot\cancel{\sin\alpha}\cdot\cos\alpha}$$

$$= \underline{\tan\alpha};$$

h)

$$1-\sin\alpha\cdot\tan\left(\frac{\alpha}{2}\right) = 1-\cancel{\sin\alpha}\cdot\frac{1-\cos\alpha}{\cancel{\sin\alpha}} = 1-(1-\cos\alpha) = \underline{\cos\alpha};$$