Übung 3

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1. Grundlegende Rechenoperationen

1.1. So weit wie möglich vereinfachen:

a) $62x - 9y - (41x - 11y) - (x - 18y)$

2.1. a) $62x - 9y - (41x - 11y) - (x - 18y) =$

$= 20x + 20y = 20(x + y)$

b) $4x - (5y + (3y - (8x - 14y + 21z) - (21y - 27z) + 23x) + 10y)$

b) $4x - (5y + (3y - (8x - 14y + 21z) - (21y - 27z) + 23x) + 10y) =$

$= 4x - (5y + 3y - 8x + 14y - 21z - 21y + 27z + 23x + 10y) =$

$= 4x - 5y - 3y + 8x - 14y + 21z + 21y - 27z - 23x - 10y =$

$= -11x - 11y - 6z$

c) $(a^2 - ab + b^2)(a + b)$

c) $(a^2 - ab + b^2)(a + b) =$

$= a^3 + a^2b - a^2b - ab^2 + ab^2 + b^3 = a^3 + b^3$

d) $((2x^2 + y^3)^2 - (2x^2 - y^3)^2)^3$

d) $((2x^2 + y^3)^2 - (2x^2 - y^3)^2)^3 =$

$= (4x^4 + 4x^2y^3 + y^6 - 4x^4 + 4x^2y^3 - y^6)^3 =$

$= (8x^2y^3)^3 = 512x^6y^9$

e) $(6x - 5y)^3 - (5y - 6x)^2 \cdot 6x$

e) $(6x - 5y)^3 - (5y - 6x)^2 \cdot 6x =$

$= (216x^3 - 540x^2y + 450xy^2 - 125y^3 - 150y^2x + 360x^2y - 216x^3)$

$= -180x^2y + 300xy^2 - 125y^3$

f) $(3x^2 - 7y^2)^2$

f) $(3x^2 - 7y^2)^2 = 9x^4 - 42x^2y^2 + 49y^4$

1.2. Brüche addieren und soweit wie möglich zusammenfassen

a) $\frac{z-1}{z}+\frac{3z^2-6z+5}{z^2}-\frac{4z^3-7z^2+5z-5}{z^3}$

a) $\frac{z-1}{z}+\frac{3z^2-6z+5}{z^2}-\frac{4z^3-7z^2+5z-5}{z^3}=$

$=\frac{z^3-z^2+3z^3-6z^2+5z-4z^3+7z^2-5z+5}{z^3}=\frac{5}{z^3}$

b) $\frac{x+6}{(x-3)^2}+\frac{x}{x^2-9}-\frac{2}{x+3}$

b) $\frac{x+6}{(x-3)^2}+\frac{x}{x^2-9}-\frac{2}{x+3}=$

$=\frac{(x+6)(x+3)+x(x-3)-2(x-3)^2}{(x-3)^2(x+3)}=$

$=\frac{x^2+9x+18+x^2-3x-2x^2+12x-18}{(x-3)^2(x+3)}=\frac{18x}{x^3-9x-3x^2+27}$

c) $\frac{a^2}{a-b}-\frac{4ab^3}{(a^2-b^2)(a+b)}-\frac{b^2(a-b)}{(a+b)^2}$

c) $\frac{a^2}{a-b}-\frac{4ab^3}{(a^2-b^2)(a+b)}-\frac{b^2(a-b)}{(a+b)^2}=$

$=\frac{a^2(a+b)(a+b)-4ab^3-b^2(a-b)(a-b}{(a-b)(a+b)(a+b)}=$

$=\frac{a^4+2a^3b+a^2b^2-4ab^3-a^2b^2+2ab^3-b^4}{(a-b)(a+b)(a+b)}=$

$=\frac{a^4+2a^3b-2ab^3-b^4}{(a-b)(a+b)(a+b)}=\frac{(a+b)^3(a-b)}{(a-b)(a+b)^2}=a+b$

1.3. So weit wie möglich vereinfachen

a) $\frac{a+b}{4a}\frac{12a}{a+b}$

a) $\frac{a+b}{4a} \cdot \frac{12a}{a+b} = 3$

b) $\left(\frac{4a}{3}+\frac{3b^2}{a^3}+\frac{b}{4a}\right)\frac{4b}{3a}$

b) $\left(\frac{4a}{3} + \frac{3b^2}{a^3} + \frac{b}{4a}\right)\frac{4b}{3a} = \frac{16a^4+36b^2+3a^2b}{12a^3} \cdot \frac{4b}{3a} =$

$= \frac{64a^4b+144b^3+12a^2b^2}{36a^4} = \frac{16a^4b+36b^3+3a^2b^2}{9a^4}$

c) $\left(\frac{a^2}{8}+\frac{a}{6}-\frac{1}{3}\right)\left(\frac{24}{a^2}-\frac{9}{a}\right)$

c) $\left(\frac{a^2}{8}+\frac{a}{6}-\frac{1}{3}\right)\left(\frac{24}{a^2}-\frac{9}{a}\right) = \frac{3a^2+4a-8}{24} \cdot \frac{24-9a}{a^2} =$

$= \frac{72a^2-27a^3+96a-36a^2+72a-192}{24a^2} = \frac{-27a^3+36a^2+168a-192}{24a} =$

$= \frac{-9a^3+12a^2+56a-64}{8a^2}$

d) $\frac{a^2+1}{a^2-1}:\frac{a+1}{a-1}$

d) $\frac{a^2+1}{a^2-1} : \frac{a+1}{a-1} = \frac{a^2+1}{(a+1)(a-1)} \cdot \frac{a-1}{a+1} =$

$= \frac{a^2+1}{a^2+2a+1}$

e) $\left(1-\frac{x-5}{x-1}\right):\left(\frac{x-1}{x-5}-1\right)$

e) $\left(1-\frac{x-5}{x-1}\right) : \left(\frac{x-1}{x-5}-1\right) =$

$= \frac{x-1-x+5}{x-1} : \frac{x-1-x+5}{x-5} =$

$= \frac{4}{x-1} \cdot \frac{x-5}{4} = \frac{x-5}{x-1}$

f) $\frac{\frac{1}{4x^2-1}+1}{x-\frac{x}{3(2x+1)}}$

$$f)\ \frac{\frac{1}{4x^2-1}+1}{x-\frac{x}{3(2x+1)}} = \frac{\frac{1+4x^2-1}{4x^2-1}}{\frac{3x(2x+1)-x}{3(2x+1)}} = \frac{4x^2}{(2x+1)(2x-1)} \cdot \frac{3\cdot(2x+1)}{x(6x+2)} =$$

$$= \frac{6x}{(3x+1)(2x-1)}$$

1.4. Geben Sie die gegebenen Terme bzw. die Ergebnisse mit positiven Exponenten an

a) $(a+b)^{-1}$

$$a)\ (a+b)^{-1} = \frac{1}{a+b}$$

b) $\frac{3x^4}{6x^{-3}}$

$$b)\ \frac{3x^4}{6x^{-3}} = \frac{x^7}{2}$$

c) $(x^2y^{-3})^{-2}(x^{-2}y^3)^2$

$$c)\ (x^2y^{-3})^{-2}(x^{-2}y^3)^2 = (x^{-4}y^6)(x^{-4}y^6) = \frac{y^{12}}{x^8}$$

d) $\left(\frac{2}{3}\right)^{-2}\left(\frac{9}{4}\right)^{-3}$

$$d)\ \left(\frac{2}{3}\right)^{-2}\left(\frac{9}{4}\right)^{-3} = \left(\frac{3}{2}\right)^2 \cdot \left(\left(\frac{3}{2}\right)^2\right)^{-3} = \left(\left(\frac{3}{2}\right)\cdot\left(\frac{3}{2}\right)^{-3}\right)^2 =$$

$$= \left(\frac{3}{2}\cdot\left(\frac{2}{3}\right)^3\right)^2 = \left(\frac{3\cdot 2^3}{2\cdot 3^3}\right)^2 = \left(\frac{4}{9}\right)^2 = \frac{16}{81}$$

1.5. Soweit wie möglich vereinfachen

a) $(3\sqrt{2}+5\sqrt{3})(3\sqrt{2}-5\sqrt{3})$

$$a)\ (3\sqrt{2}+5\sqrt{3})(3\sqrt{2}-5\sqrt{3}) =$$

$$= (3\sqrt{2})^2 - (5\sqrt{3})^2 = 18 - 75 = -57$$

b) $\sqrt[5]{x}:\sqrt{x}$

$$b)\ \sqrt[5]{x} : \sqrt{x} = \frac{x^{\frac{1}{5}}}{x^{\frac{1}{2}}} = x^{\left(\frac{1}{5}-\frac{1}{2}\right)} = x^{-\frac{3}{10}} = \frac{1}{\sqrt[10]{x^3}}$$

1.6. Nenner rational machen

a) $\frac{\sqrt{x^2-y^2}}{\sqrt{x+y}}$

a $\frac{\sqrt{x^2-y^2}}{\sqrt{x+y}} = \frac{\sqrt{x^2-y^2}\cdot\sqrt{x+y}}{x+y} = \frac{\sqrt{(x+y)(x-y)}\cdot\sqrt{(x+y)}}{x+y} = \sqrt{x-y}$

b) $\frac{\sqrt[3]{x^2-2x+1}}{\sqrt[3]{x-1}}$

b $\frac{\sqrt[3]{x^2-2x+1}}{\sqrt[3]{x-1}} = \frac{\sqrt[3]{(x-1)^2}\sqrt[3]{(x-1)^2}}{x-1} = \sqrt[3]{x-1}$

1.7. Terme so weit wie möglich zerlegen

a) $\log\frac{ab}{cd}$

a) $\log\frac{ab}{cd} = \log ab - \log cd = \log a + \log b - \log c - \log d$

b) $\log\frac{a^2+b^2}{a^2-b^2}$

b) $\log\frac{a^2+b^2}{a^2-b^2} = \log(a^2+b^2) - \log(a^2-b^2) =$

$= \log(a^2+b^2) - \log(a+b) - \log(a-b)$

c) $\log\frac{x^3}{y^2}\sqrt[4]{\frac{a^2x}{3b^3}}$

c) $\log\frac{x^3}{y^2}\sqrt[4]{\frac{a^2x}{3b^3}} = 3\log x - 2\log y + \frac{1}{4}\log a^2x - \frac{1}{4}\log 3b^3$

$= 3\log x - 2\log y + \frac{1}{2}\log a + \frac{1}{4}\log x - \frac{1}{4}\log 3 - \frac{3}{4}\log b =$

$= 3\frac{1}{4}\log x - 2\log y + \frac{1}{2}\log a - \frac{3}{4}\log b - \frac{1}{4}\log 3$

d) $\log \frac{x^9 \sqrt[5]{(z-y)^4 y^3}}{y^7 \sqrt[7]{(z-y)^5 x^{12}}} (xz^2)^5$

d) $\log \frac{x^9 \sqrt[5]{(z-y)^4 y^3}}{y^7 \sqrt[7]{(z-y)^5 x^{12}}} (xz^2)^5 =$

$= \log x^9 \sqrt[5]{(z-y)^4 y^3} - \log y^7 \sqrt[7]{(z-y)^5 x^{12}} + \log (xz^2)^5 =$

$= 9\log x + \frac{1}{5}\log (z-y)^4 y^3 - 7\log y - \frac{1}{7}\log (z-y)^5 x^{12} + 5\log xz^2 =$

$= 9\log x + \frac{4}{5}\log(z-y) + \frac{3}{5}\log y - 7\log y - \frac{5}{7}\log(z-y) - \frac{12}{7}\log x + 5\log x + 10\log z$

$= 12\frac{2}{7}\log x + \frac{3}{35}\log(z-y) - 6\frac{2}{5}\log y + 10\log z$

y 42 : 8 = 5 R 2   8 · 0,5 = 4 + 0,0

5 : 8 = 0 R 5

y = 52.4

z 42 : 16 = 2 R 10   16 · 0,5 = 8 + 0,0

2 : 16 = 0 R 2

z = 2A . 8