<u>Lösung Funktionsgraphen</u>

Damit ihr euch auch selbst Funktionen plotten könnt, benutzt einfach einen Online Plotter wie folgenden:

https://www.mathe-fa.de/de#result

a) $f(x) = 2x + 5$

b) $g(x) = x - 5$

c) $h(x) = -3x$

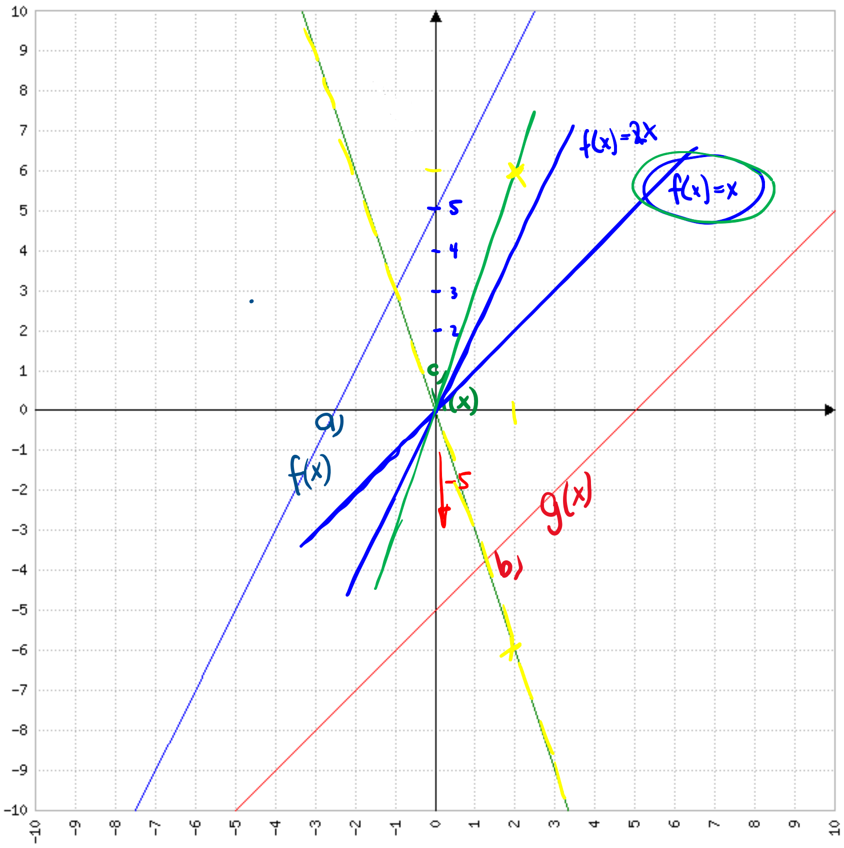
Bedeutung: Spiegelung an der x-Achse

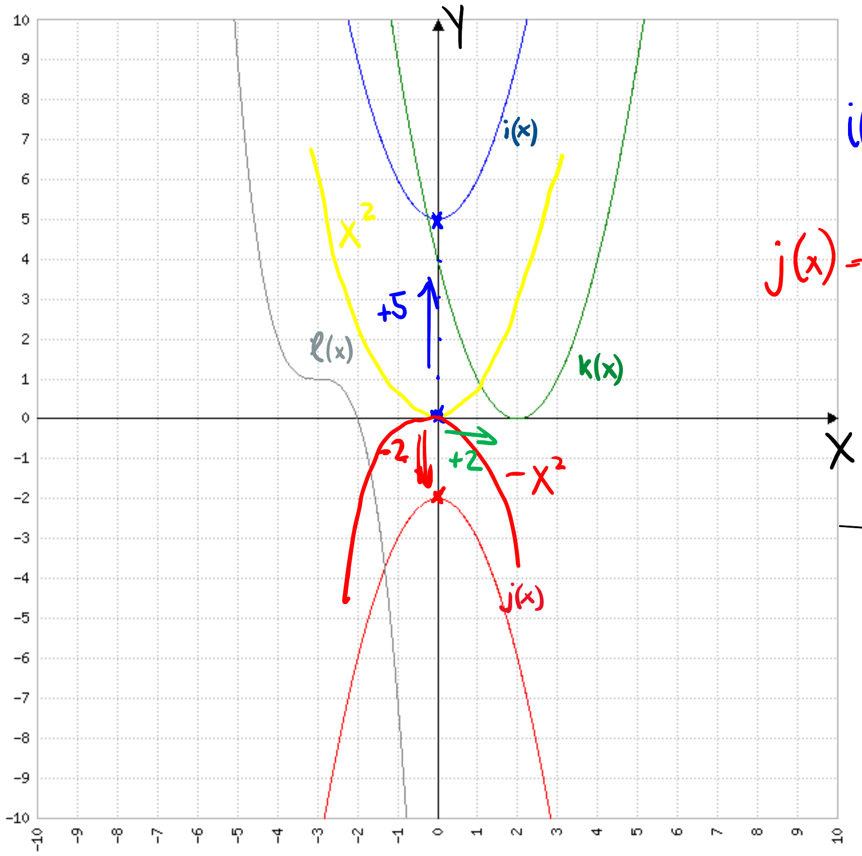
d) $i(x) = x^2 + 5$

e) $j(x) = -x^2 - 2$

f) $k(x) = (x-2)^2$

g) $l(x) = -(x+3)^3 + 1$






$i(x) = x^2 + 5$

$= y$-Achsen-abschnitt

$j(x) = -x^2 - 2$

$(x-2)^2 \rightarrow$ Verschiebung in x-Richtung

$\cancel{(x+3)^3+2}$

Parabelfunktion

$(ax-b)^2 + c$

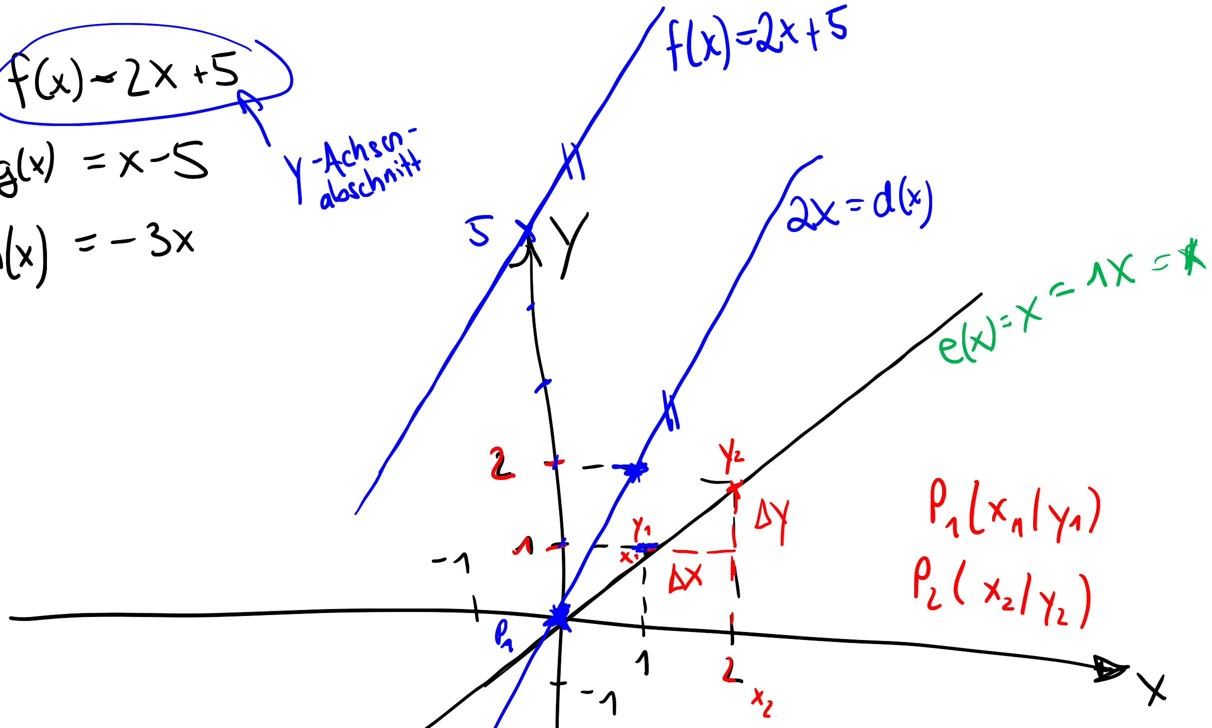
$K(x) = (x+2)^2 - 3$

a, $f(x) = 2x + 5$

$g(x) = x - 5$

$h(x) = -3x$

Y-Achsenabschnitt

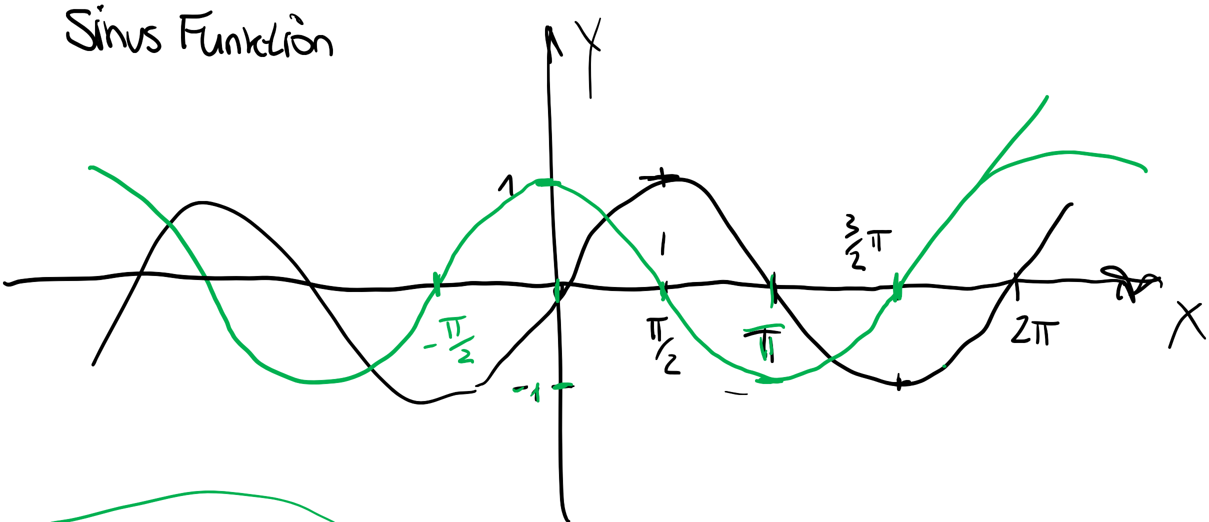



Steigung $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-1}{2-1} = \frac{1}{1} = 1$

$m = 2 = \frac{\Delta y}{\Delta x}$

$P_1(0|0)$ $P_2(1/2)$

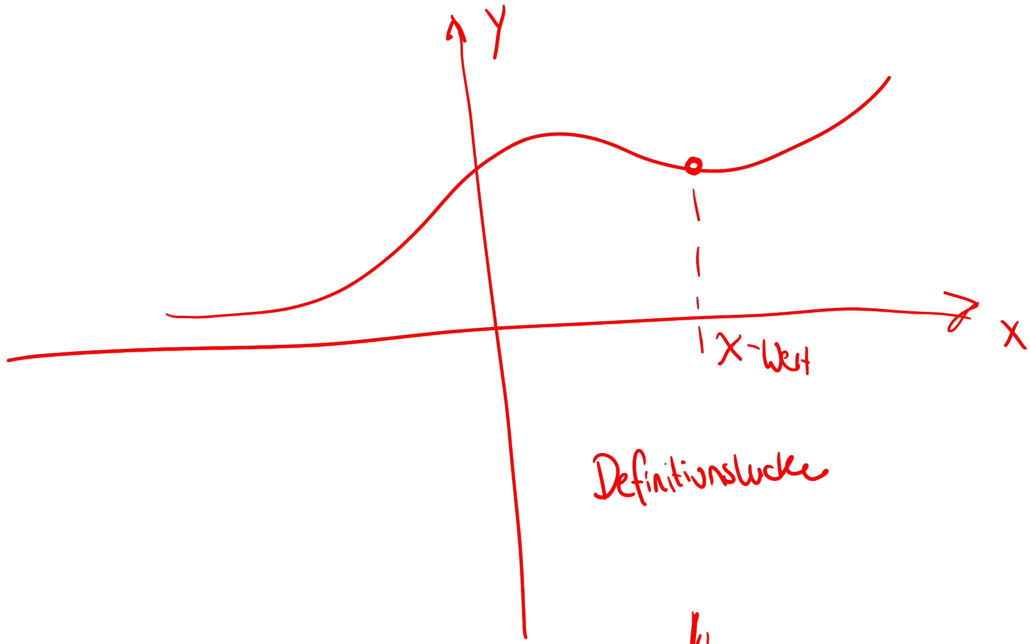
Sinus Funktion

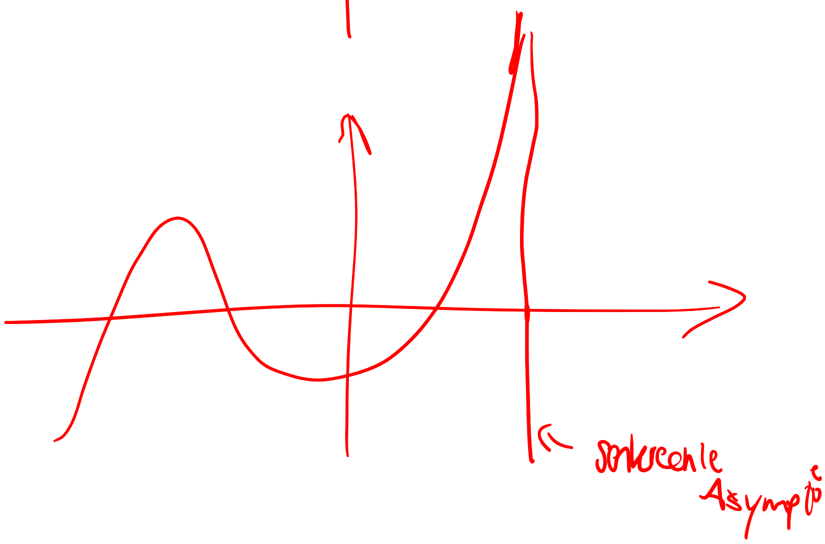



Cosinus Funktion

~~$\cos(x) = \sin x + \frac{\pi}{2}$~~ falsch!

$\cos(x) = \sin\left(x + \frac{\pi}{2}\right)$




$$f(x) = \frac{x-1}{(x+2)(x-3)^2}$$

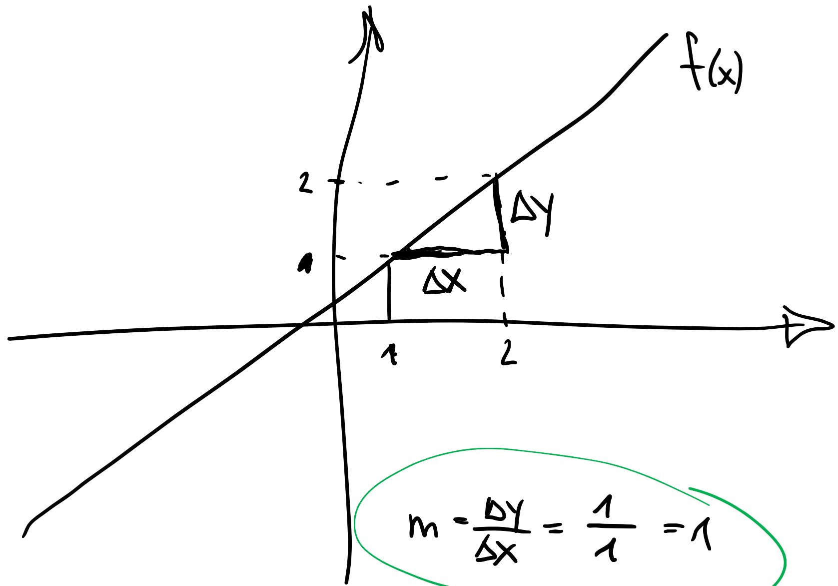
$x = 1$, ist eine Nullstelle

Definitionslücken = Polstellen: $x_2 = -2$ $x_3 = 3$

einfache doppelte Polstelle

Nullstelle $0 = x - 1 \rightarrow x = 1$

Polstelle $0 = \underbrace{(x+2)}_{-2}\underbrace{(x-3)^2}_{3}$

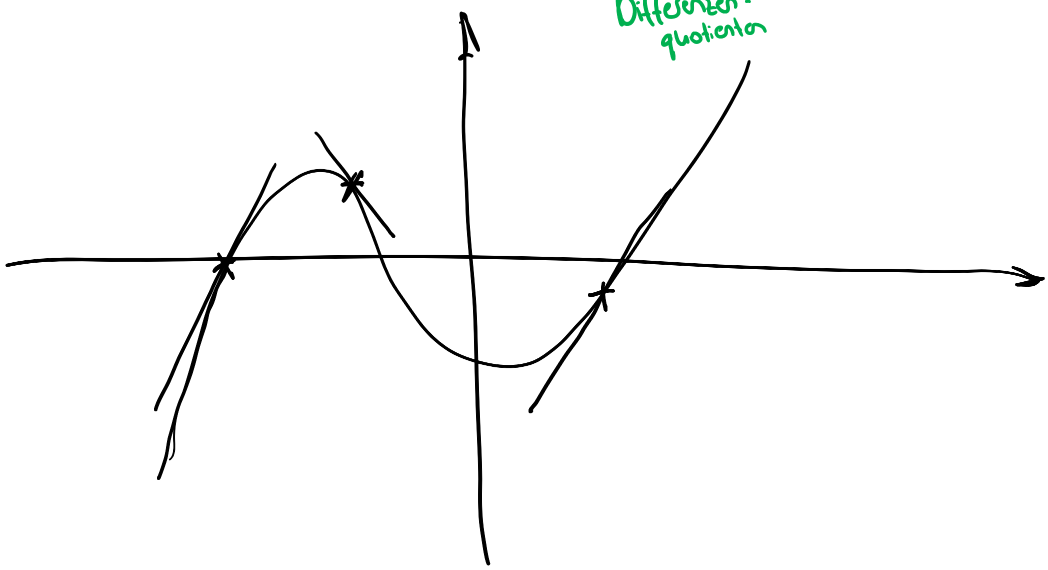


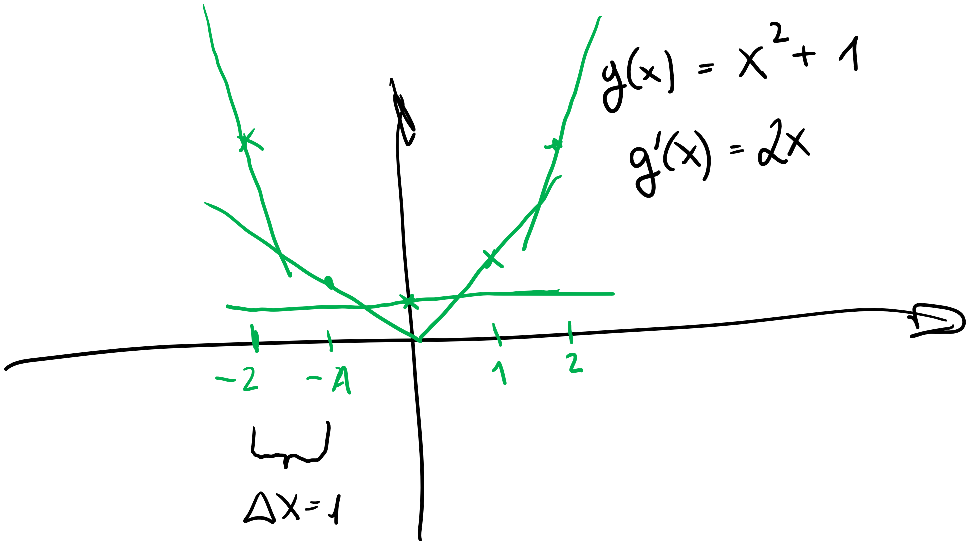

$f(x) = x + 0{,}5$

$f'(x) = 1$

$m = \frac{\Delta y}{\Delta x} = \frac{1}{1} = 1$

Differenzen-quotienten

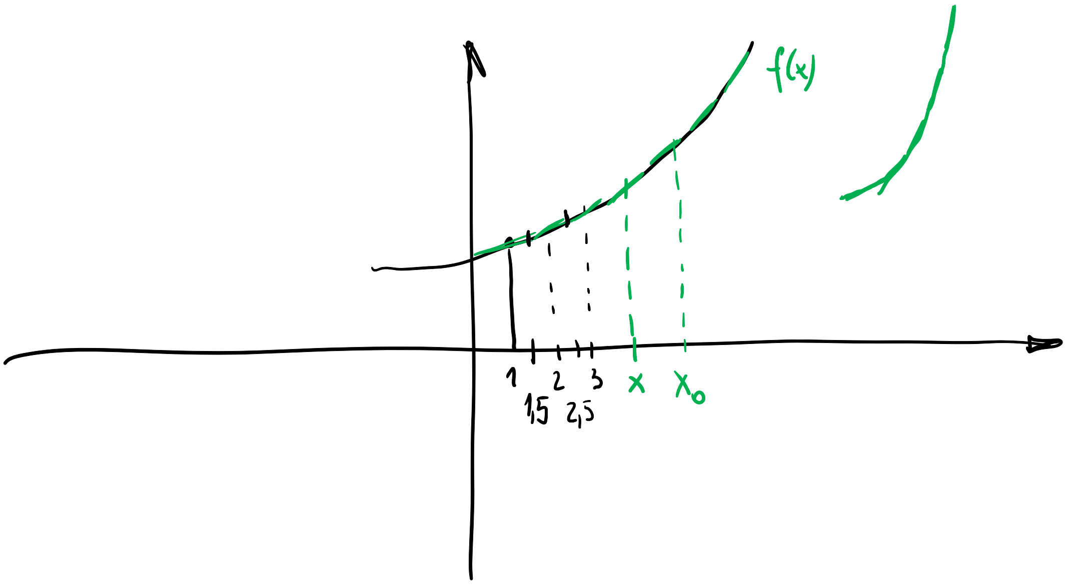





$g(x) = x^2 + 1$

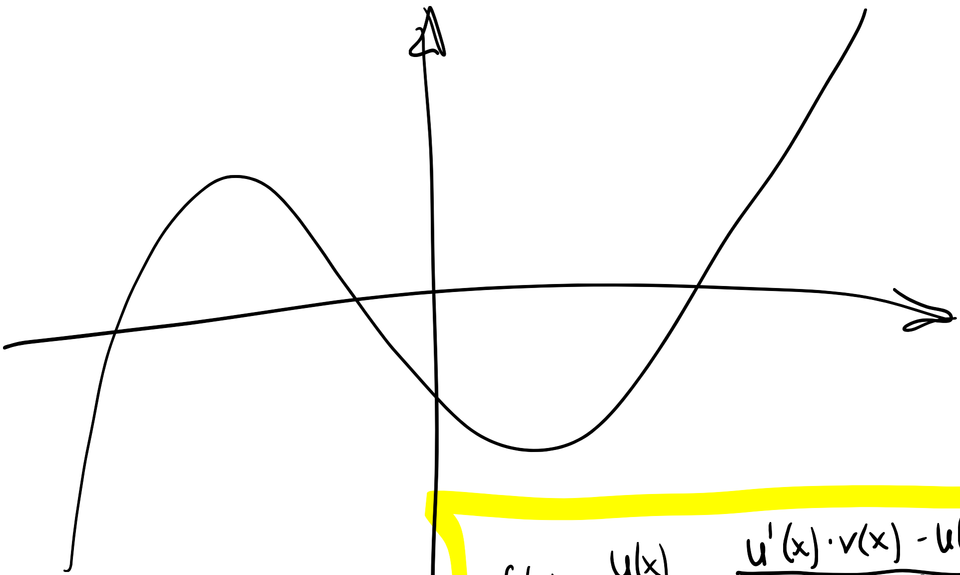
$g'(x) = 2x$

$\Delta x \to 0$




$$f'(x_0) = \lim_{x \to x_0} \frac{f(x)-f(x_0)}{x-x_0}$$

momentane Änderungsrate



$$\boxed{f(x) = \frac{u(x)}{v(x)} = \frac{u'(x)\cdot v(x) - u(x)\cdot v'(x)}{[v(x)]^2}}$$

a) $f(x) = \dfrac{x^2-2x+1}{x-1}$ $\quad u(x) = x^2-2x+1 \quad u'(x) = 2x-2$

$v(x) = x-1 \quad v'(x) = 1$

$$f'(x) = \frac{(2x-2)(x-1) - ((x^2-2x+1)\cdot 1)}{(x-1)^2} = \frac{2x^2-2x-\cancel{2x}+2-x^2+\cancel{2x}-1}{(x-1)^2}$$

$$= \frac{x^2-2x+1}{(x-1)^2} = \frac{(x-1)^2}{(x-1)^2} = 1$$

$$f(x) = \frac{x^2-2x+1}{x-1} = \frac{(x-1)^2}{x-1} = x-1$$

$$f'(x) = 1$$

d) $f(x) = \dfrac{4x^2+12x+9}{3x+3}$ $\quad u(x) = 4x^2+12x+9 \quad u'(x) = 8x+12$

$v(x) = 3x+3 \quad v'(x) = 3$

$$f'(x) = \frac{(8x+12)(3x+3) - (4x^2+12x+9)\cdot(3)}{(3x+3)^2}$$

$$= \frac{24x^2+36x+24x+36 - 12x^2 - 36x - 27}{(3x+3)^2}$$

$$= \underline{\underline{\frac{12x^2+24x+9}{(3x+3)^2}}} = \frac{12x^2+24x+9}{9x^2+18x+9} \qquad \frac{12x^2+36x+9}{(3x+3)^2}$$

5,

b, $f(x) = \sqrt{\frac{3+x}{3-x}}$ Ketteregel: $f(x) = u(v(x))$ $\quad f'(x) = u'(v(x))\cdot v'(x)$

$f(x) = \left(\frac{3+x}{3-x}\right)^{\frac{1}{2}}$ $\quad u(x) = 3+x \quad u'(x) = 1$

$\quad v(x) = 3-x \quad v'(x) = -1$

$$f'(x) = \frac{1}{2}\cdot\left(\frac{3+x}{3-x}\right)^{-\frac{1}{2}}\cdot\left[\frac{1\cdot(3-x) - (3+x)\cdot(-1)}{(3-x)^2}\right]$$

$$= \frac{1}{2}\cdot\left(\frac{3+x}{3-x}\right)^{-\frac{1}{2}}\cdot\frac{3-\cancel{x}+3+\cancel{x}}{(3-x)^2}$$

$$= \frac{6}{2\cdot(3-x)^2}\cdot\left(\frac{3+x}{3-x}\right)^{-\frac{1}{2}} = \frac{3}{(3-x)^2}\cdot\frac{1}{\left(\frac{3+x}{3-x}\right)^{\frac{1}{2}}} = \frac{3}{(3-x)^2}\cdot\frac{1}{\sqrt{\frac{3+x}{3-x}}}$$

c, $$f(x) = \frac{(\overset{a}{x^2} + \overset{2\cdot a\cdot c}{4x} + \overset{=c^2}{4})^2}{x+2} = \frac{((x+2)^2)^2}{x+2} = \frac{(x+2)^4}{(x+2)^1} = (x+2)^3$$

$$\Downarrow$$

$$(x+2)^{4-1} = \underline{\underline{(x+2)^3}}$$

$$f'(x) = 3\cdot(x+2)^2\cdot 1 = 3(x+2)^2$$

g) $f(x) = \frac{(x+1)^3}{x}$ $\quad u(x) = (x+1)^3 \quad u'(x) = 3(x+1)^2\cdot 1$

$\quad v(x) = x \quad v'(x) = 1$

$$f'(x) = \frac{3(x+1)^2\cdot x - (x+1)^3\cdot 1}{x^2} = \frac{3(x+1)^2\cdot x - (x+1)^3}{x^2}$$

$$= \frac{3\cdot(x^2+2x+1)\cdot x - [(x^2+2x+1)(x+1)]}{x^2} = \frac{3x^3+6x^2+3x - \boxed{[x^3 - x^2 - 2x^2 - 2x - x - 1]}}{x^2}$$

$$= \frac{2x^3+3x^2-1}{x^2}$$