a) $2\ln\left(\frac{p}{q}\right) + \frac{1}{2}\ln(q^2) - \ln(p^3) =$

$2\cdot(\ln p - \ln q) + \frac{1}{2}\cdot 2\cdot \ln q - 3\cdot \ln p =$

$= 2\ln p - 2\ln q + \ln q - 3\ln p =$

$= -\ln p - \ln q = \ln(-p) - \ln q = \ln\left(\frac{-p}{q}\right)$ $\quad \ln 1 = 0$

$\left(\ln\left(\frac{1}{pq}\right) \Rightarrow \ln 1 - (\ln p + \ln q) = -\ln p - \ln q\right)$

$= -\ln p - \ln q = -(\ln p + \ln q) = -\ln(pq)$

$= 0 - \ln(pq) = \ln 1 - \ln(pq) = \ln\left(\frac{1}{pq}\right)$

$\ln(e)^2 = 2\cdot \ln e = 2\cdot \underbrace{\log_e e}_{=1} = 2\cdot 1 = 2$ ✓

$e^{\ln(3)} = 3$ ✓ $\qquad \log_a b = c \qquad a^c = b$

$e^{\ln(e)} = e$ ✓ $\qquad \lg b = \log_{10} b$

$\ln b = \log_e b$

$e^c = b$

$e^{\ln b} = e^{\log_e b} = b$

$\ln\left(\frac{1}{\sqrt{e}}\right) = \ln 1 - \ln\sqrt{e} = 0 - \ln e^{\frac{1}{2}} = -\frac{1}{2}$ ✓

$\ln(\sqrt{e}) = \ln e^{\frac{1}{2}} = \frac{1}{2}\cdot \underbrace{\ln e}_{=1} = \frac{1}{2}$

$\ln(3e^k) = \ln 3 + \ln e^k = \ln 3 + k\cdot \underbrace{\ln e}_{=1}$

$= \ln 3 + k$ ✓

→ besondere Schreibweisen:

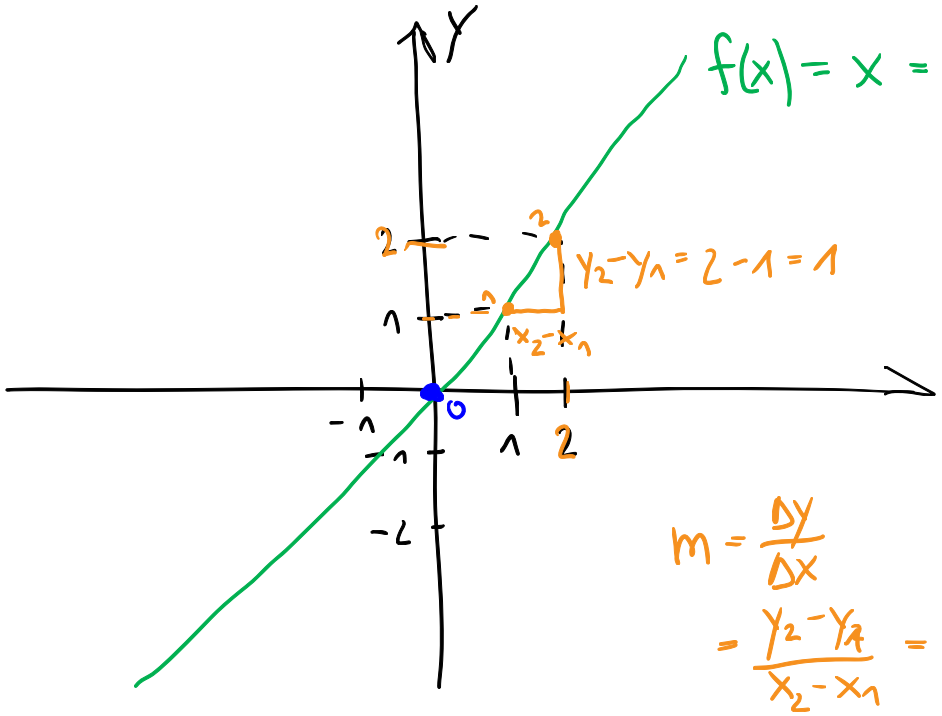
(logarithmus binomialis)

$\log_2 x = lb$

$\log_{10} x = \lg x$

$\log_e x = \ln x$ ← naturalis

Eulerzahl $\approx 2{,}718$



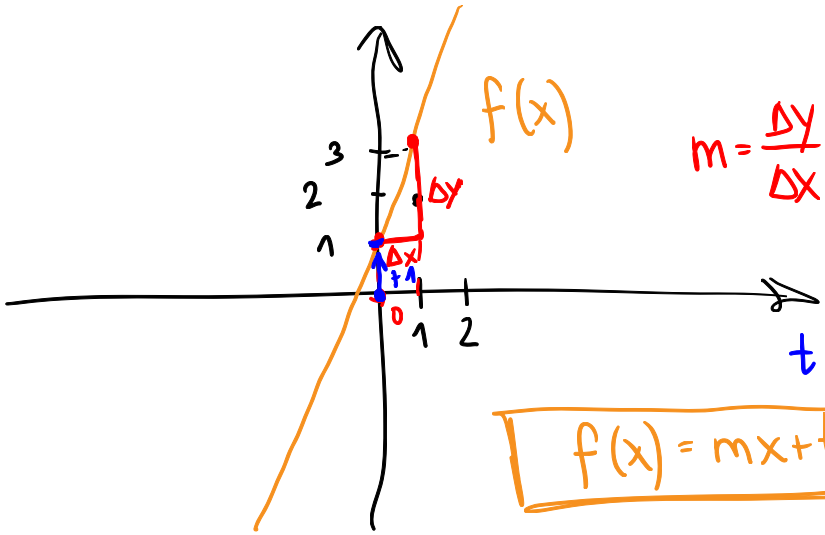
$f(x) = x = 1 \cdot x + 0$

$f(x) = mx + t$

↑ Steigung ↑ y-Achsenabschnitt

Δ = Delta

$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1}{2-1} = \frac{1}{1} = 1$



$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-1}{1-0} = \frac{2}{1}$

$= 2$

$t = +1$

$f(x) = mx + t = 2x + 1$