

$$1) \cos^2(x) \cdot \tan^2(x) + \cos^2(x) = ?$$

$$\boxed{\cos^2(x) \neq \cos(x^2)} \quad \tan^2(x) = (\tan(x))^2$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\cos^2(x) \cdot \left(\frac{\sin(x)}{\cos(x)} \right)^2 + \cos^2(x) =$$

$$\text{mit } \left(\frac{a}{b} \right)^r = \frac{a^r}{b^r}$$

$$= \cancel{\cos^2(x)} \cdot \frac{\sin^2(x)}{\cancel{\cos^2(x)}} + \cos^2(x) =$$

$$= \sin^2(x) + \cos^2(x) = 1$$

$$h, \quad 1 - \sin(\alpha) \cdot \tan\left(\frac{\alpha}{2}\right)$$

$$\tan \frac{\alpha}{2} = \frac{\sin \alpha}{1 + \cos \alpha} = \frac{1 - \cos \alpha}{\sin \alpha}$$

$$1 - \cancel{\sin \alpha} \cdot \frac{1 - \cos \alpha}{\cancel{\sin \alpha}}$$

$$1 - (1 - \cos \alpha) = 1 - 1 + \cos \alpha = \underline{\underline{\cos \alpha}}$$

4. Leiten Sie mit Hilfe des Additionstheorems $\cos(\alpha + \beta) = \cos(\alpha) \cdot \cos(\beta) - \sin(\alpha) \cdot \sin(\beta)$ die Gültigkeit der Beziehung

$$\cos(2\alpha) = 2 \cdot \cos^2(\alpha) - 1$$

her!

$$\cos(\alpha + \beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$$

$$\cos(\alpha + \beta) = \frac{1}{2} [\cancel{\cos(\alpha - \beta)} + \cos(\alpha + \beta)] - \frac{1}{2} [\cancel{\cos(\alpha - \beta)} - \cos(\alpha + \beta)]$$

$$\cos(\alpha + \beta) = \frac{1}{2} [\cos(\alpha + \beta) - (-\cos(\alpha + \beta))]$$

$$\cos(\alpha + \beta) = \frac{1}{2} (2 \cdot \cos(\alpha + \beta))$$

$\cos\alpha +$

$$2) \frac{1 - \cos^2(x)}{\sin(x) \cos(x)} =$$

~~$\cos$~~

$$\cos(\alpha + \beta) + \cos(\alpha - \beta) = 2 \cdot \cos\alpha \cdot \cos\beta \quad | - \cos(\alpha - \beta)$$

$$\cos(\alpha + \beta) = 2 \cdot \cos\alpha \cdot \cos\beta - \cos(\alpha - \beta)$$

$$2 \cdot \cos\alpha \cdot \cos\beta - \cos(\alpha - \beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta \quad | - \cos\alpha \cdot \cos\beta$$

$$\cos\alpha \cdot \cos\beta - \cos(\alpha - \beta) = -\sin\alpha \cdot \sin\beta \quad \text{Formel 7.6.6 Nr.1}$$

$$\cos\alpha \cdot \cos\beta - \cos(\alpha - \beta) = -\left(\frac{1}{2}(\cos(\alpha - \beta) - \cos(\alpha + \beta))\right)$$

$$\cos\alpha \cdot \cos\beta - \cos(\alpha - \beta) = -\frac{1}{2}\cos(\alpha - \beta) + \frac{1}{2}\cos(\alpha + \beta) \quad | + \frac{1}{2}\cos(\alpha - \beta)$$

$$\cos\alpha \cos\beta - \frac{1}{2}\cos(\alpha - \beta) = \frac{1}{2}\cos(\alpha + \beta)$$

Aufgabe 4

Annahme $\alpha = \beta$

$$2) \quad \frac{1 - \cos^2(x)}{\sin(x) \cos(x)} =$$

Mit $1 = \sin^2 x + \cos^2 x$

$$\frac{\sin^2 x + \cos^2 x - \cos^2 x}{\sin x \cdot \cos x} = \frac{\sin^2 x}{\sin x \cdot \cos x} = \frac{\sin x}{\cos x} = \underline{\underline{\tan x}}$$

$$3) \quad \frac{\sin(x+y) + \sin(x-y)}{\cos(x+y) + \cos(x-y)} =$$

Trigonometrische Formeln

$$\sin(A \pm B) = \sin A \cdot \cos B \pm \cos A \cdot \sin B$$

$$\cos(A \pm B) = \cos A \cdot \cos B \mp \sin A \cdot \sin B$$

$$\frac{\sin x \cdot \cos y + \cancel{\cos x \cdot \sin y} + \sin x \cdot \cos y - \cancel{\cos x \cdot \sin y}}{\cos x \cdot \cos y - \cancel{\sin x \cdot \sin y} + \cos x \cdot \cos y + \cancel{\sin x \cdot \sin y}} =$$

$$= \frac{2 \cdot \sin x \cdot \cos y}{2 \cdot \cos x \cdot \cos y} = \frac{\sin x}{\cos x} = \underline{\underline{\tan x}}$$

$$4) \frac{1 - \cos(2x)}{\sin(2x)} =$$

$$\frac{1 - \cos^2 x + \sin^2 x}{2 \cdot \sin x \cdot \cos x}$$