Ingenieurmathematik 1 – Studiengang Systems Engineering
WS 2016/2017 Dr. Andreas Hiemer

Übung 8

1. Differenzialrechnung

1.1. Bilden Sie die erste Ableitung

a) $y = x^7$ $\quad y' = 7x^6$

b) $y = x^{-4}$ $\quad y' = -4x^{-5}$

c) $y = \frac{1}{x^{-4}}$ $\quad y' = 4x^3$

d) $y = x^{-\frac{12}{7}}$ $\quad y' = -\frac{12}{7} x^{-\frac{19}{7}}$

e) $y = \frac{1}{\sqrt{x}}$ $\quad y' = -\frac{1}{2} x^{-\frac{3}{2}} = -\frac{1}{2x\sqrt{x}}$

f) $y = \sqrt[4]{x}\sqrt[3]{x}$ $\quad y = x^{\frac{1}{4}} \cdot x^{\frac{1}{3}} = x^{\frac{7}{12}}$ $\quad y' = \frac{7}{12} \cdot x^{-\frac{5}{12}}$

g) $y = \sqrt{x\sqrt{x}}$ $\quad y = \sqrt{x\sqrt{x}} = \sqrt{x^{\frac{3}{2}}} = x^{\frac{3}{4}}$ $\quad y' = \frac{3}{4} x^{-\frac{1}{4}}$

h) $y = \frac{\sqrt[6]{x^5}\sqrt[3]{x^4}}{\sqrt[12]{x^{11}}}$

$$y = \frac{x^{\frac{5}{6}} \cdot x^{\frac{4}{3}}}{x^{\frac{11}{12}}} = x^{\frac{10}{12} + \frac{16}{12} - \frac{11}{12}} = x^{\frac{15}{12}} = x^{\frac{5}{4}}$$

$$y' = \frac{5}{4} \cdot x^{\frac{1}{4}} = \frac{5\sqrt[4]{x}}{4}$$

i) $y = 3x^{-3}$ $\quad y' = -9x^{-4}$

j) $y = \frac{3}{x^2}$ $\quad y' = -\frac{6}{x^3}$

k) $y = 7\sqrt[7]{x^5}$ $\quad y' = \frac{5}{7} \cdot 7 \cdot x^{-\frac{2}{7}} = 5 \cdot \frac{1}{\sqrt[7]{x^2}} = \frac{5}{\sqrt[7]{x^2}}$

l) $y = 17\sqrt[25]{2x^{-1}}$ $\quad y' = -\frac{17}{25x} \, ?\sqrt[25]{\frac{2}{x}}$

m) $y = 4x^5 - 3x^4 + 12x^3 + 7x^2 + 3x - 1$

$y' = 20x^4 - 12x^3 + 36x^2 + 14x + 3$

1.2. Berechnen Sie $f'(x_0)$

a) $y = 3x^2 - 5x + 8 \qquad x_0 = \frac{1}{12}$

$y' = 6x - 5 \qquad y'|_{x_0} = y'\left(\frac{1}{12}\right) = -4{,}5$

b) $y = 8x^5 - 3x^4 + 2x^3 - 3x^2 + 8x - 17 \qquad x_o = -1$

$y' = 40x^4 - 12x^3 + 6x^2 + 8 - 6x$

$y'|_{x_0} = 40 + 12 + 6 + 8 + 6 = 72$

c) $y = \sqrt[4]{5x^3} - \sqrt[5]{4x^4} + \sqrt[6]{5x^5} - 5x^{-\frac{1}{5}} + 8x^{\frac{1}{3}} \qquad x_0 = 1$

$y' = \sqrt[4]{5} \cdot \frac{3}{4} \cdot x^{-\frac{1}{4}} - \frac{4}{5}\sqrt[5]{4}\, x^{-\frac{1}{5}} + \sqrt[6]{5}\,\frac{5}{6}\, x^{-\frac{1}{6}} + x^{-\frac{6}{5}} + \frac{8}{3} x^{-\frac{2}{3}}$

$y'|_{x_0} = \frac{3}{4}\sqrt[4]{5} - \frac{4}{5}\sqrt[5]{4} + \frac{5}{6}\sqrt[6]{5} + 1 + \frac{8}{3} =$

$= \frac{3}{4}\sqrt[4]{5} - \frac{4}{5}\sqrt[5]{4} + \frac{5}{6}\sqrt[6]{5} + \frac{11}{3}$

1.3. Berechnen Sie $f'(x)$

a) $y = (x-5)(2x+2)$

$f'(x) = 1(2x+2) + (x-5)\cdot 2 = 4x - 8$

b) $y = (2x+4)(2x+4)$

$f'(x) = 2(2x+4) + (2x+4)\cdot 2 = 8x + 16$

c) $y = (3x^2 - 2x + 1)(9x^2 + x - 1)$

$f'(x) = (6x-2)(9x^2+x-1) + (3x^2-2x+1)(18x+1) =$

$= 54x^3 - 12x^2 - 8x + 2 + 54x^3 - 33x^2 + 16x + 1 =$

$= 108x^3 - 45x^2 + 8x + 3$

d) $y = \frac{x+1}{x-1}$

$f'(x) = \frac{1\cdot(x-1) - (x+1)\cdot 1}{(x-1)^2} = -\frac{2}{(x-1)^2}$

e) $y = \frac{4x^2-1}{3x^2+5}$

$f'(x) = \frac{8x\cdot(3x^2+5) - (4x^2-1)\cdot 6x}{(3x^2+5)^2} = \frac{24x^3 + 40x - 24x^3 + 6x}{(3x^2+5)^2} =$

$= \frac{46x}{(3x^2+5)^2}$

f) $y=\dfrac{x^2-6x+9}{x^2-9}$

$$f(x)=\frac{x^2-6x+9}{x^2-9}=\frac{(x-3)^2}{(x+3)(x-3)}=\frac{x-3}{x+3}$$

$$f'(x)=\frac{1\cdot(x+3)-(x-3)\cdot 1}{(x+3)^2}=\frac{6}{(x+3)^2}$$

g) $y=\tan^2 x$

$$y'=\frac{2}{\cos^2 x}\cdot\tan x=\frac{2\sin x}{\cos^3 x}$$

h) $y=\ln x^3$

$$y'=\frac{1}{x^3}\cdot 3x^2=\frac{3}{x}$$

i) $y=\dfrac{\sin x\cos x}{x^2}$

$$y'=\frac{(\cos x\cdot\cos x+\sin x\cdot(-\sin x))\cdot x^2-\sin x\cdot\cos x\cdot 2x}{x^4}$$

$$=\frac{x\cdot\cos 2x-\sin 2x}{x^3}$$

j) $y=(\ln x)^2x^3$

$$y'=2\ln x\cdot\frac{1}{x}\cdot x^3+(\ln x)^2\cdot 3x^2=$$

$$=2x^2\ln x+3x^2(\ln x)^2$$

k) $y=(4-8x)^8$

$$y'=8\cdot(4-8x)^7\cdot(-8)=-64(4-8x)^7$$

l) $y=(x^9-1)^{-2}$

$$y'=-2(x^9-1)^{-3}\cdot 9x^8=-18x^8\cdot(x^9-1)^{-3}$$

m) $y=(x-1)^{\frac{1}{4}}$

$$y'=\frac{1}{4}(x-1)^{-\frac{3}{4}}=\frac{1}{4\sqrt[4]{(x-1)^3}}$$

n) $y=(x^3-4)^8$

$$y'=8(x^3-4)^7\cdot 3x^2=24x^2(x^3-4)^7$$

o) $y=\left(x-\dfrac{3}{x}\right)^2$

$$y'=2\left(x-\frac{3}{x}\right)\cdot\left(1+\frac{3}{x^2}\right)=2x\left(1-\frac{9}{x^4}\right)$$

p) $y=\sqrt{x-1}$

$$y' = \frac{1}{2}\cdot(x-1)^{-\frac{1}{2}} = \frac{1}{2\sqrt{x-1}}$$

q) $y=\left(\frac{x^5-1}{x+3}\right)^3$

$$y' = 3\left(\frac{x^5-1}{x+3}\right)^2\cdot\left(\frac{5x^4(x+3)-(x^5-1)}{(x+3)^2}\right) =$$

$$= 3\left(\frac{x^5-1}{x+3}\right)^2\cdot\frac{4x^5+15x^4+1}{(x+3)^2} = \frac{3(x^5-1)^2(4x^5+15x^4+1)}{(x+3)^4}$$

r) $y=\frac{\sqrt{x^2+5}}{2x-13}$

$$y' = \frac{\frac{1}{2}(x^2+5)^{-\frac{1}{2}}\cdot 2x\cdot(2x-13)-(x^2+5)^{\frac{1}{2}}\cdot 2}{(2x-13)^2} =$$

$$= \frac{(x^2+5)^{-\frac{1}{2}}\left[(2x^2-13x)-(x^2+5)\cdot 2\right]}{(2x-13)^2}$$

$$= -\frac{13x+10}{\sqrt{x^2+5}\,(2x-13)^2}$$

1.4. Berechnen Sie $f'(x_0)$

a) $y=(x-2)(4x^2+3x+6);\ x_0=\frac{1}{4}$

$$y' = 4x^2+3x+6+(x-2)(8x+3) = 12x^2-10x$$

$$y'\Big|_{x_0=\frac{1}{4}} = \frac{12}{16}-\frac{10}{4} = -\frac{7}{4}$$

b) $y=\sqrt{x^3}(x-2)^2 \qquad x_0=4$

$$y' = \frac{1}{2}\left(x^3\right)^{-\frac{1}{2}} \cdot 3x^2 \cdot (x-2)^2 + \sqrt{x^3} \cdot 2(x-2) =$$

$$= \frac{1}{2\sqrt{x^3}} \cdot 3x^2 (x-2)^2 + 2(x-2)\sqrt{x^3} =$$

$$= \frac{3x}{2\sqrt{x}}(x-2)^2 + 2x(x-2)\sqrt{x}$$

$$y'\Big|_{x_0=4} = \frac{12}{4} \cdot 4 + 8 \cdot 2 \cdot 2 = 44$$

c) $y=\dfrac{2x-3}{x-5} \qquad x_0=-2$

$$y' = \frac{2x-10-2x+3}{(x-5)^2} = -\frac{7}{(x-5)^2}$$

$$y'\Big|_{x_0=-2} = -\frac{7}{49} = -\frac{1}{7}$$

d) $y=\dfrac{6+5x+x^2}{(x+3)^2}$ $\quad x_0=-1$

$$y'=\frac{(5+2x)(x+3)^2-(6+5x+x^2)(2x+6)}{(x+3)^4}=$$

$$=\frac{(5+2x)(x+3)-(6+5x+x^2)\cdot 2}{(x+3)^3}$$

$$y'\Big|_{x_0=-1}=\frac{3\cdot 2-2\cdot 2}{2^3}=\frac{1}{4}$$

e) $y=\dfrac{2-3x}{2+x}-\dfrac{1+4x}{1-x}-\dfrac{x^2-14x}{x^2+x-2}$ $\quad x_0=3$

$$y'=\frac{-6-3x-2+3x}{(2+x)^2}-\frac{4-4x+1+4x}{(1-x)^2}-\frac{(2x-14)(x^2+x-2)-(x^2-14x)(2x+1)}{(x^2+x-2)^2}=$$

$$=\frac{-8}{(2+x)^2}-\frac{5}{(1-x)^2}-\frac{2x^3+2x^2-4x-14x^2-14x+28-2x^3-x^2+28x^2+14x}{(x^2+x-2)^2}=$$

$$=\frac{-8}{(2+x)^2}-\frac{5}{(1-x)^2}-\frac{15x^2-4x+28}{(x^2+x-2)^2}$$

$$y'\Big|_{x_0=3}=-\frac{8}{25}-\frac{5}{4}-\frac{151}{100}=-\frac{308}{100}$$

1.5. Berechnen Sie $f'(x)$

a) $y=\dfrac{x}{x-\sqrt{9+x^2}}$

$$y'=\frac{x-\sqrt{9+x^2}-x\left(1-\frac{1}{2}(9+x)^{-\frac{1}{2}}\cdot 2x\right)}{\left(x-\sqrt{9+x^2}\right)^2}=$$

$$=\frac{x-\sqrt{9+x^2}-x+x^2(9+x)^{-\frac{1}{2}}}{\left(x-\sqrt{9+x^2}\right)^2}=\frac{-9}{\left(x-\sqrt{9+x^2}\right)^2\cdot\sqrt{9+x^2}}$$

b) $y=\sin 2x$

$$y'=2\cos 2x$$

c) $y=\sin(3+4x)$

$$y'=4\cos(3+4x)$$

d) $y = \frac{1}{\sin 2x}$

$y' = -\frac{2\cos 2x}{\sin^2 2x}$

e) $y = \frac{2}{\sqrt{\sin x}}$

$y' = 2 \cdot \left(-\frac{1}{2}\right) \sin^{-\frac{3}{2}} x \cdot \cos x = -\frac{\cos x}{\sqrt[3]{\sin^3 x}} = -\frac{\cos x}{\sin x \sqrt{\sin x}}$

f) $y = \cos x^2$

$y' = -\sin x^2 \cdot 2x$

g) $y = \cos^2 \sqrt{x}$

$y' = 2\cos\sqrt{x} \cdot (-\sin\sqrt{x}) \cdot \frac{1}{2\sqrt{x}} = -\frac{\sin 2\sqrt{x}}{2\sqrt{x}}$

h) $y = \tan 9x$

$y' = \frac{9}{\cos^2 9x}$

i) $y = \tan(5-7x)$

$y' = -\frac{7}{\cos^2(5-7x)}$

j) $y = \frac{5}{\sqrt{3\tan x}}$

$y' = -\frac{5}{2} \cdot (3\tan x)^{-\frac{3}{2}} \cdot \frac{1 \cdot 3}{\cos^2 x} = -\frac{5 \cdot 3}{2\sqrt{(3\tan x)^3} \cdot \cos^2 x} =$

$= -\frac{5 \cdot 3}{6\tan x \sqrt{3\tan x} \cdot \cos^2 x} = -\frac{5}{2\sin x \sqrt{3\tan x}\cos x} = \frac{5}{\sin 2x \sqrt{3\tan x}}$

1.6. Berechnen Sie $f'(x_0)$

a) $y = \ln(3x+2)^3$ $\qquad x_0 = 1$

$y = (\ln(3x+2))^3 = 3\ln(3x+2)$

$y' = \frac{3}{3x+2} \cdot 3 = \frac{9}{3x+2}$

$y'\Big|_{x_0=1} = \frac{9}{5}$

b) $y = \ln \frac{x+1}{x-1}$ $x_o = 2$

$$y = \ln \frac{x+1}{x-1} = \ln(x+1) - \ln(x-1)$$

$$y' = \frac{1}{x+1} - \frac{1}{x-1}$$

$$y'\Big|_{x_0=2} = \frac{1}{3} - 1 = -\frac{2}{3}$$

c) $y = \ln \sqrt{\frac{x+1}{x-1}}$ $x_0 = 2$

$$y = \ln \sqrt{\frac{x+1}{x-1}} = \frac{1}{2}\left(\ln(x+1) - \ln(x-1)\right)$$

$$y' = \frac{1}{2}\left(\frac{1}{x+1} - \frac{1}{x-1}\right)$$

$$y'\Big|_{x_0=2} = -\frac{1}{3}$$

d) $y = \ln \sqrt{\frac{2+3x}{2-3x}}$ $x_0 = \frac{\sqrt{2}}{3}$

$$y = \ln \sqrt{\frac{2+3x}{2-3x}} = \frac{1}{2}\left(\ln(2+3x) - \ln(2-3x)\right)$$

$$y' = \frac{1}{2}\left(\frac{3}{2+3x} + \frac{3}{2-3x}\right)$$

$$y'\Big|_{x_0=\frac{\sqrt{2}}{3}} = \frac{1}{2}\left(\frac{3}{2+\sqrt{2}} + \frac{3}{2-\sqrt{2}}\right) = 3$$