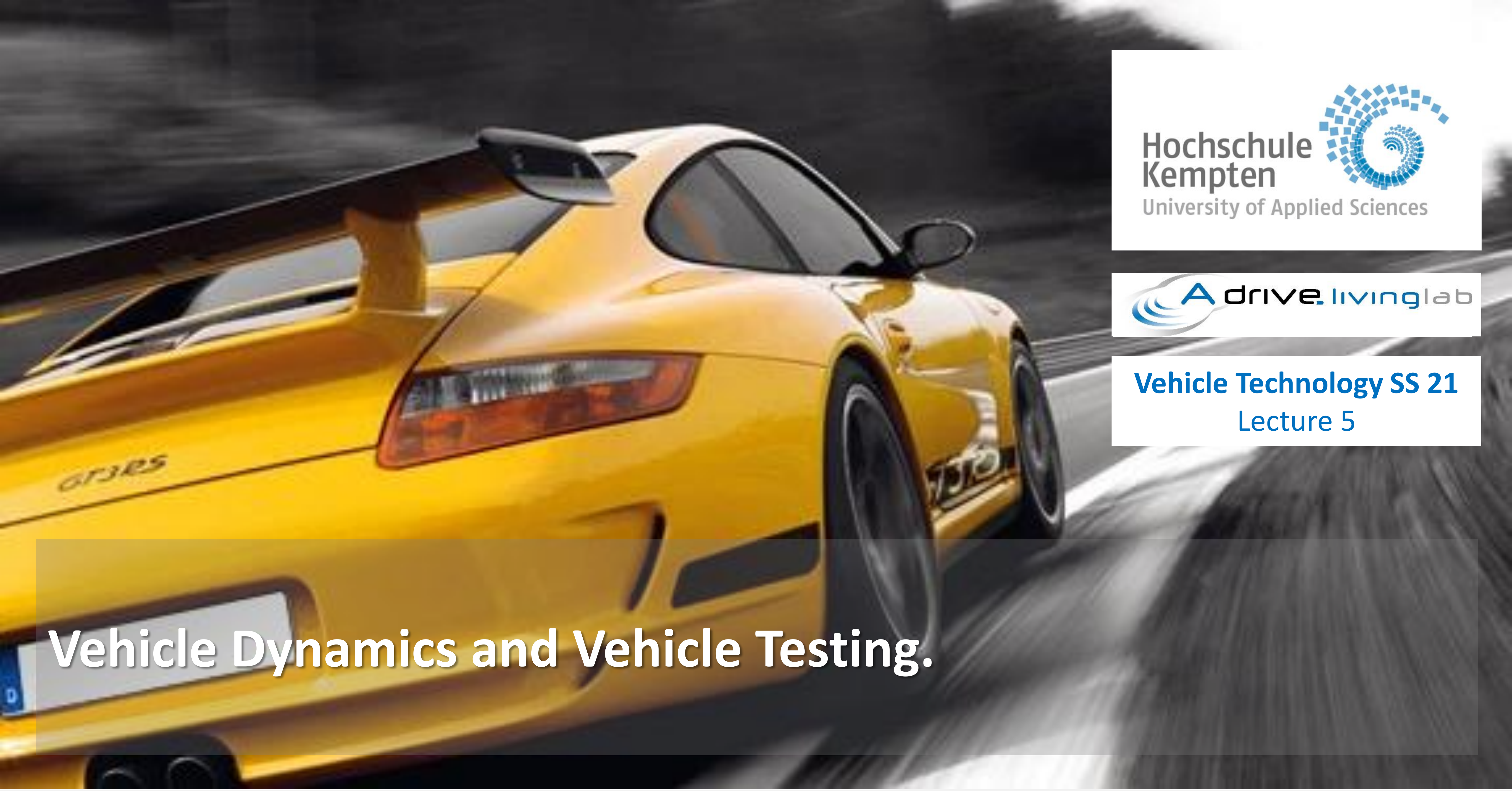


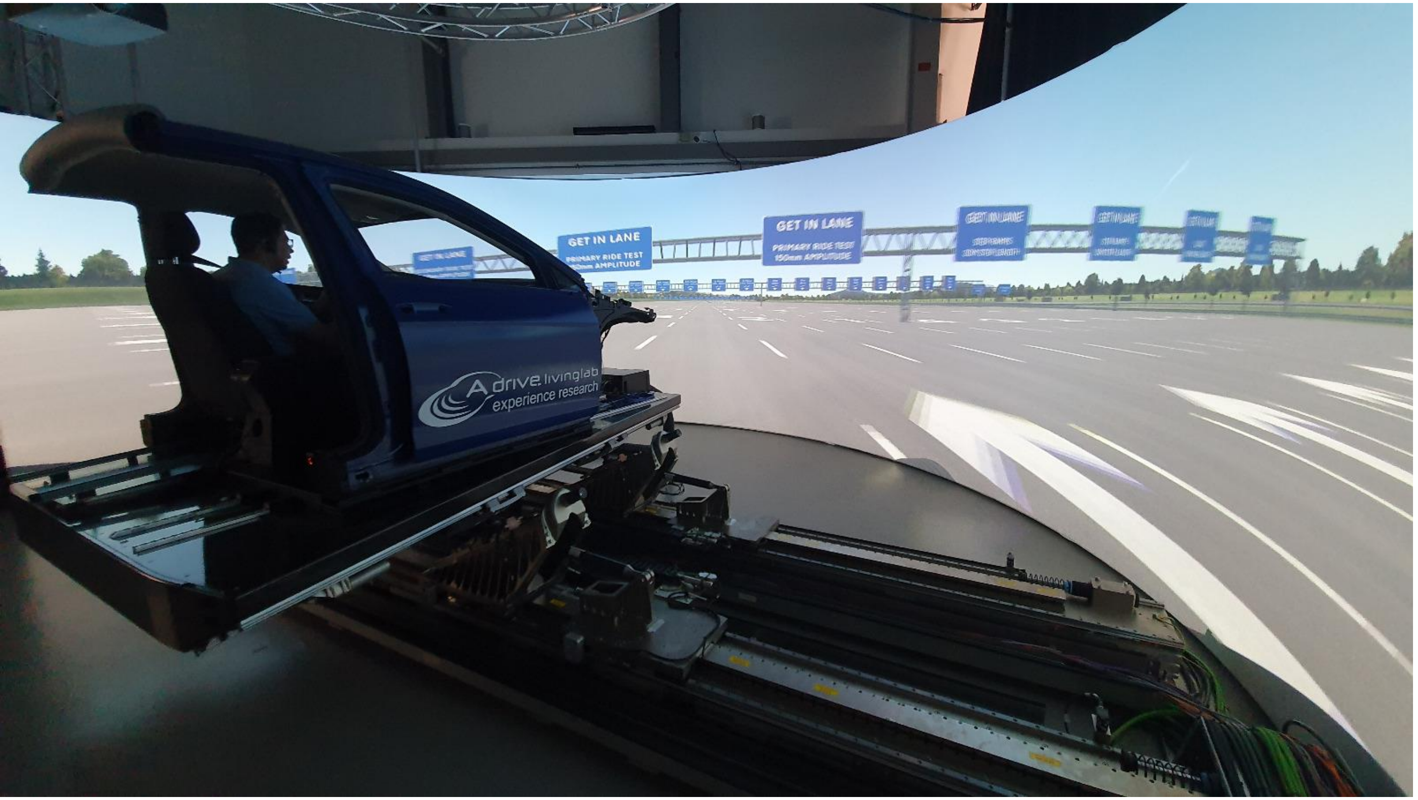
Hochschule Kempten
University of Applied Sciences

Adrive.livinglab

Vehicle Technology SS 21
Lecture 5

Vehicle Dynamics and Vehicle Testing.

Basic vehicle dynamics calculation and vehicle models



Maximilian Böhle, M. Sc. Automotive Engineering

maximilian.boehle@hs-kempten.de

A drive livinglab

Kempten University of Applied Sciences

Basic vehicle dynamics calculation and vehicle models

What is a model?

A simplified representation of the reality.

What is a vehicle dynamics model?

Refers to calculate the dynamic motion of ground vehicles for engineering tasks.

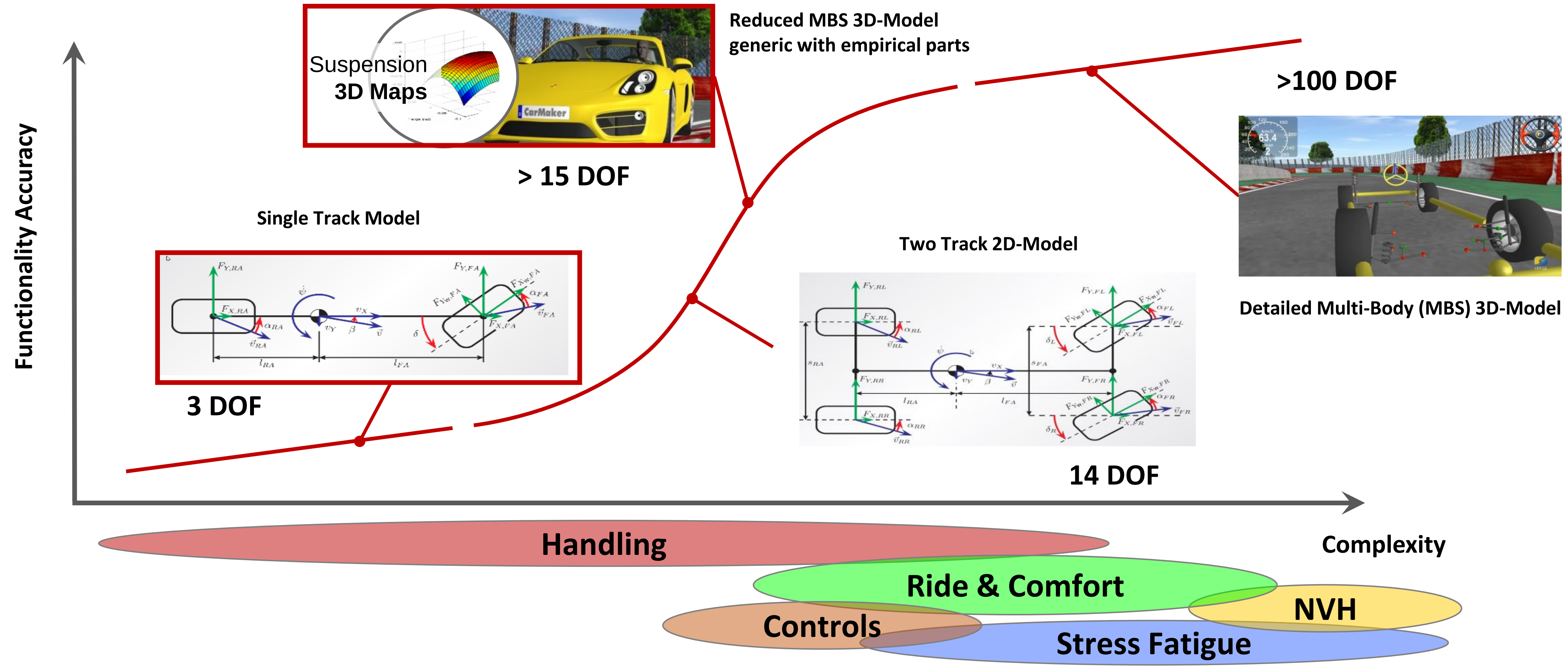
Where do we need vehicle dynamics models?

Simulation, model based testing and model based control methods.

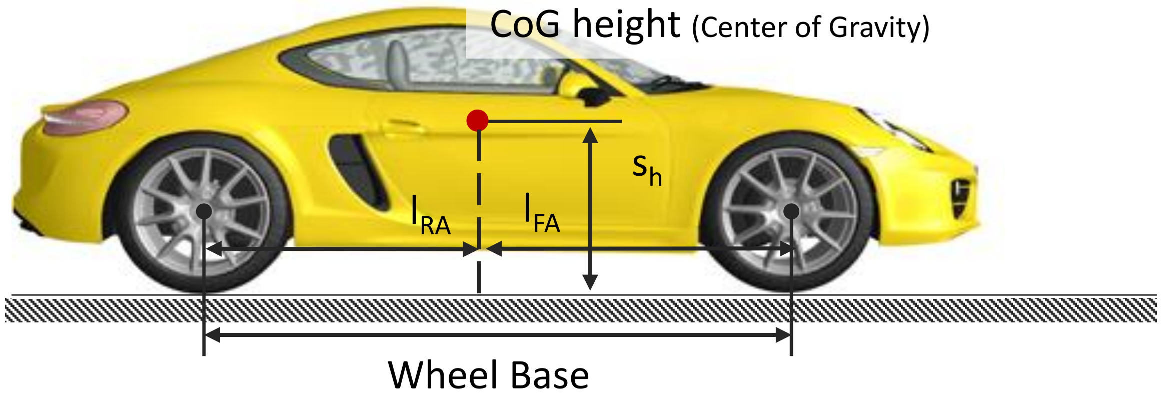
When is a vehicle dynamics model valid?

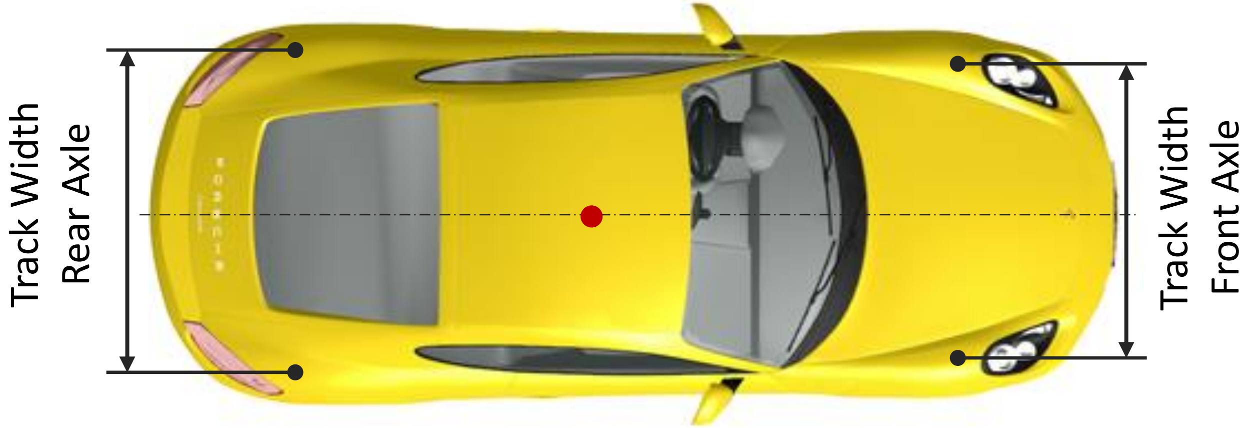
Good and accurate enough for the application purpose?

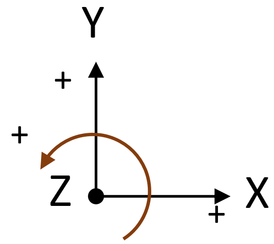
Classification an positioning of vehicle dynamics models



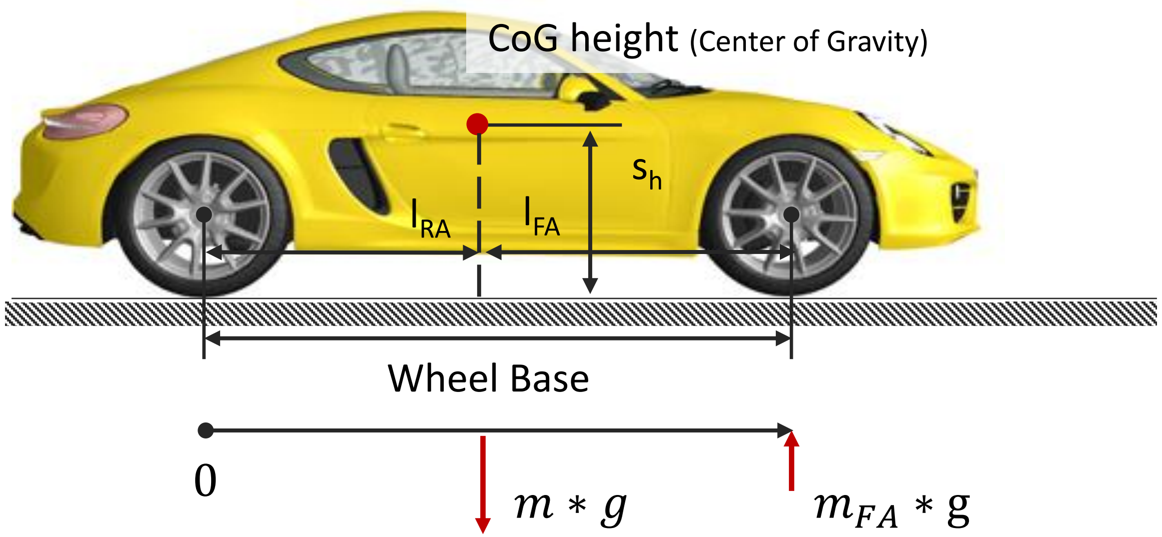
Basic chassis dimensions

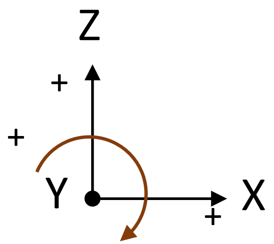






Calculation of CoG – Center of Gravity





$$\sum F_z = 0$$

$$m * g * l_{RA} = m_{FA} * g * l$$

$$l_{FA} = l - l_{RA}$$

- Vehicle weight (with driver) = 1.970 kg
- m_{FA} (front axle) = 1100 kg
- m_{RA} (rear axle) = 870 kg
- Wheel base = 2.807 mm,
- Center of gravity = 0,65 m

Calculation of CoG – Center of Gravity

$$\boxed{m * \cancel{g} * l_{RA} = m_{FA} * \cancel{g} * l}$$

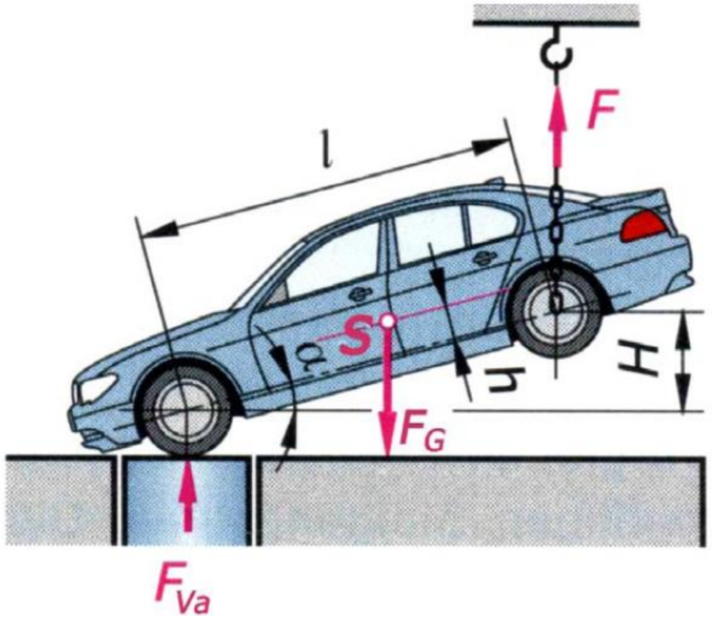
$$\boxed{\boldsymbol{l_{RA}} = \frac{m_{FA} * l}{m}} = \frac{1100\,kg * 2{,}807\,m}{1970\,kg} = \mathbf{1{,}567\ m}$$

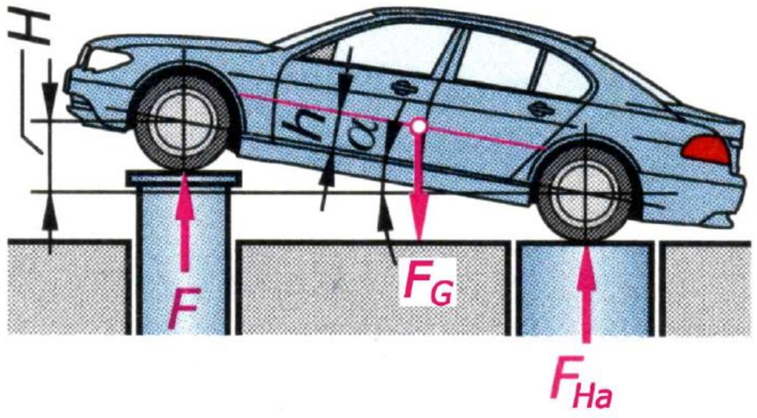
$$\boxed{l_{FA} = l - l_{RA}}$$

$$\boxed{\boldsymbol{l_{FA}} = l - l_{RA} = 2{,}807m - 1{,}567m = 1{,}24\,\boldsymbol{m}}$$

- Vehicle weight (with driver) = 1.970 kg
- m_{FA} (front axle) = 1100 kg
- m_{RA} (rear axle) = 870 kg
- Wheel base = 2.807 mm,
- Center of gravity = 0,65 m

Calculation of CoG – Center of Gravity





$$h = l * \frac{F_{FA} - F}{F_G * tan\alpha}$$

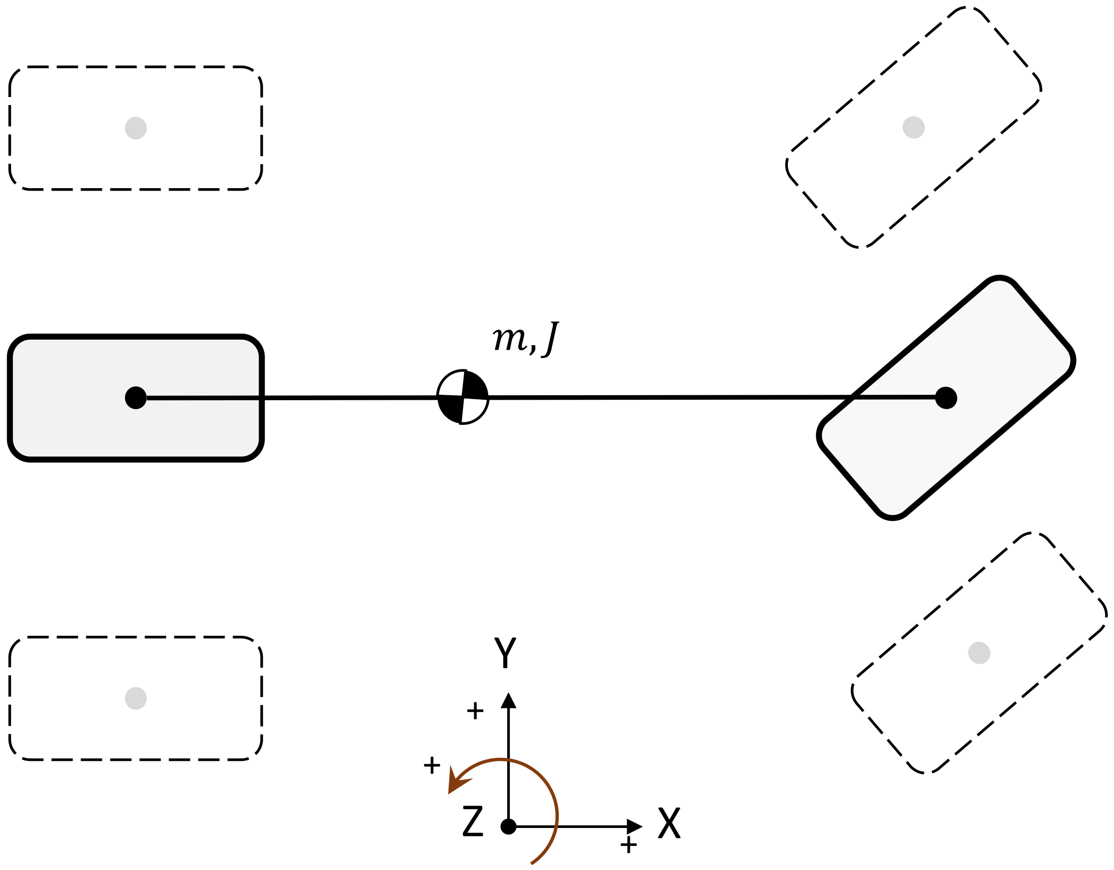
$$h = l * \frac{F_{HA} - F}{F_G * tan\alpha}$$

Calculation of CoG – Center of Gravity

[2]

[3]

Simplification of the vehicle model: "Single Track Model" Theory - 3 DOF



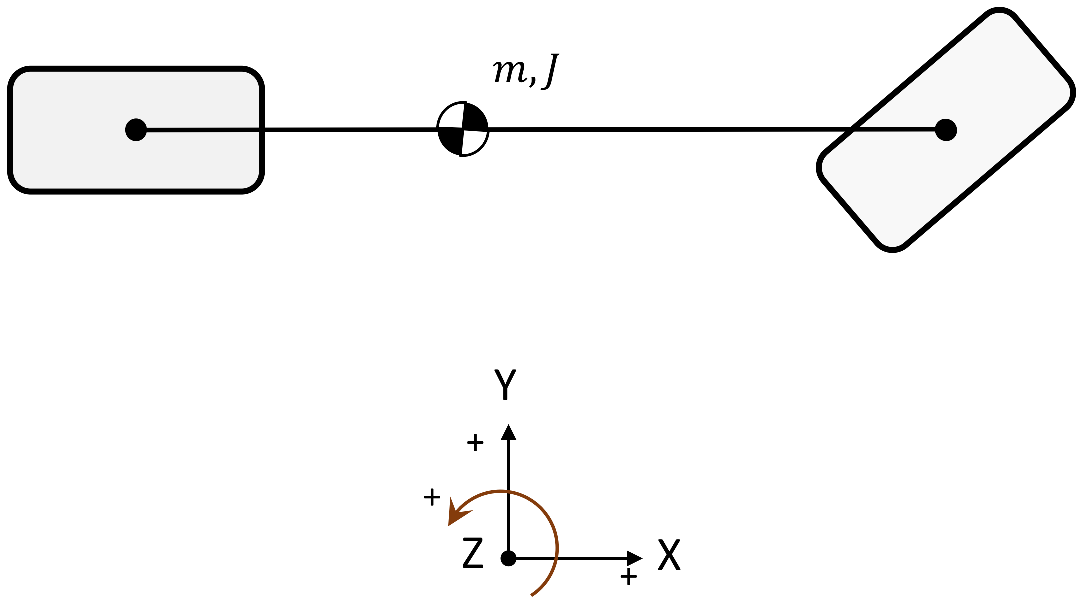
Approach

- **Wheels are lumped into single track**
  → Tire side slip & axle stiffness are combined per axle
- Rigid body with CoG in-plane (on-track)
- Only horizontal movement
  → No roll, pitch & vertical motion
- Steering angle only at the front axle

3 Degrees of Freedom (DOF)

- Longitudinal
- Lateral
- Yaw (rotation around Z)

Simplification of the vehicle model: “Single Track Model” Theory - 3 DOF



<u>Approach</u>

- **Wheels are lumped into single track**
  → Tire side slip & axle stiffness are combined per axle

- Equations of motion based on:
  - Geometrical equation
  - Equilibrium of forces & moments
  - Transversal system stiffness (Tire / Axle)

<u>**3 Degrees of Freedom** (DOF)</u>

- Longitudinal
- Lateral
- Yaw (rotation around Z)

Notations

m	vehicle mass
F_x, F_y	Forces
F_{x_w}, F_{y_w}	Wheel forces
v	vehicle speed
v_x, v_y	velocity longitudinal / lateral
a_x, a_y	acceleration longitudinal / lateral
v_{FA}, v_{RA}	velocity front / rear axle
l	wheel base
l_{FA}, l_{RA}	length front / rear axle to center of gravity
α_{FA}, α_{RA}	Side slip angle front / rear axle
c_{FA}, c_{RA}	side slip stiffness front / rear axle
β	drift angle
$\dot{\beta}$	drift angle speed

δ	steering angle (at wheel)
δ_H	steering wheel angel (SWA)
i_s	steering ratio
ψ	yaw angle
$\dot{\psi}$	yaw angle speed
R, r	course radius
FA, RA	Index front axle, rear axle
$stat$	index for stationary
CoG	center of gravity
EG	understeer gradient (Eigenlenkgradient)

Basic vehicle dynamics calculation and vehicle models

Lateral Motion **(1)**

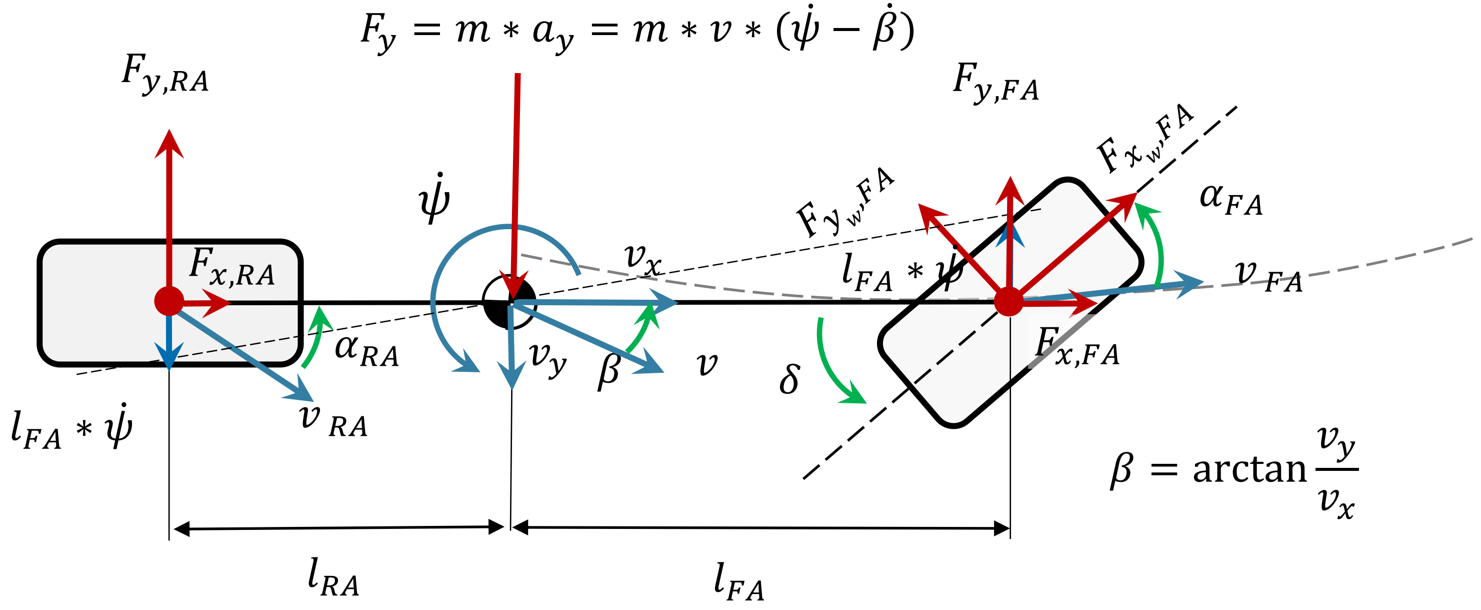
$$F_y = F_{y,FA} + F_{y,RA} = m * a_y = m * v * (\dot{\psi} - \dot{\beta})$$

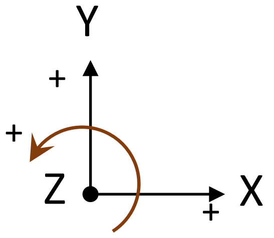
$$\sum F_y = 0$$

Longitudinal Motion **(2)**

$$F_x = F_{x,FA} + F_{x,RA} = m * a_x$$

$$\sum F_x = 0$$





Rotation Z-Axle **(3)**

$$\theta * \ddot{\psi} = F_{y,FA} * l_{FA} - F_{y,RA} * l_{RA}$$

$$\sum M_z = 0$$

Basic vehicle dynamics calculation and vehicle models

Lateral Motion (1)

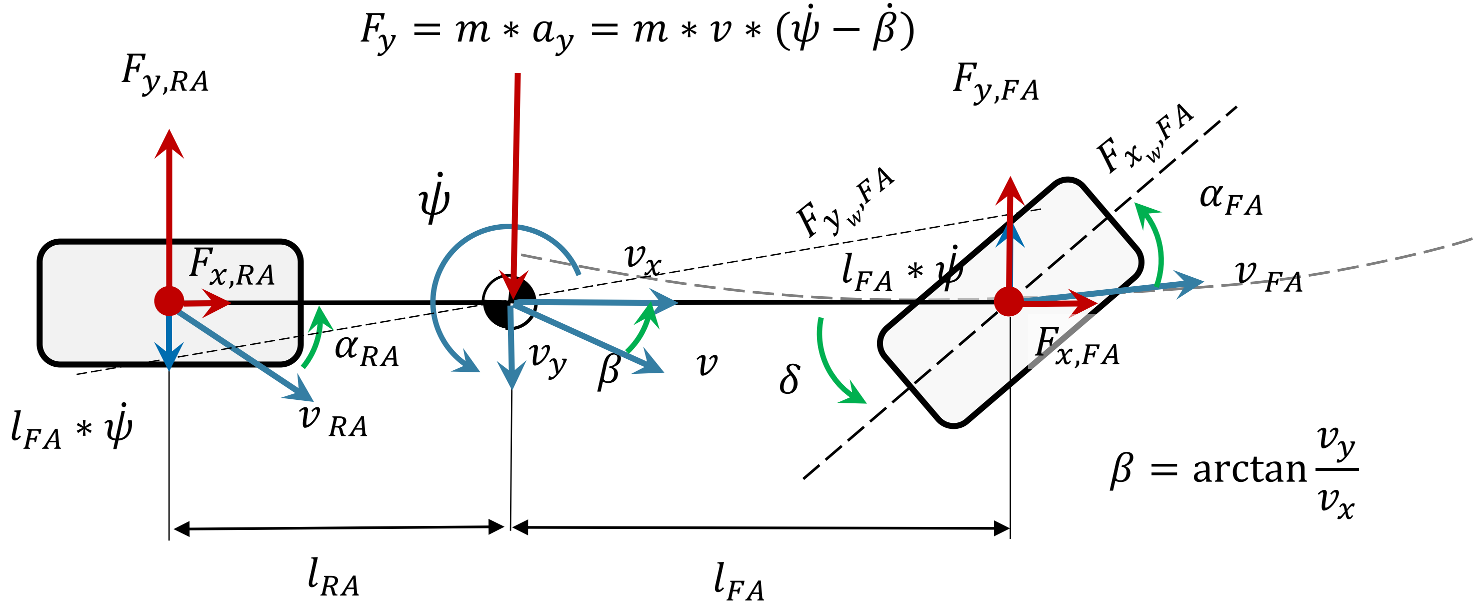
$$F_y = F_{y,FA} + F_{y,RA} = m * a_y = m * v * (\dot{\psi} - \dot{\beta})$$

$$\sum F_y = 0$$

Longitudinal Motion (2)

$$F_x = F_{x,FA} + F_{x,RA} = m * a_x$$

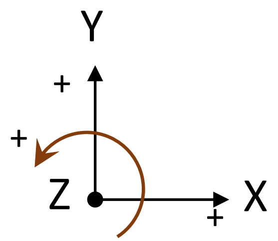
$$\sum F_x = 0$$



Rotation Z-Axle (3)

$$\theta * \ddot{\psi} = F_{y,FA} * l_{FA} - F_{y,RA} * l_{RA}$$

$$\sum M_z = 0$$



Lateral Motion

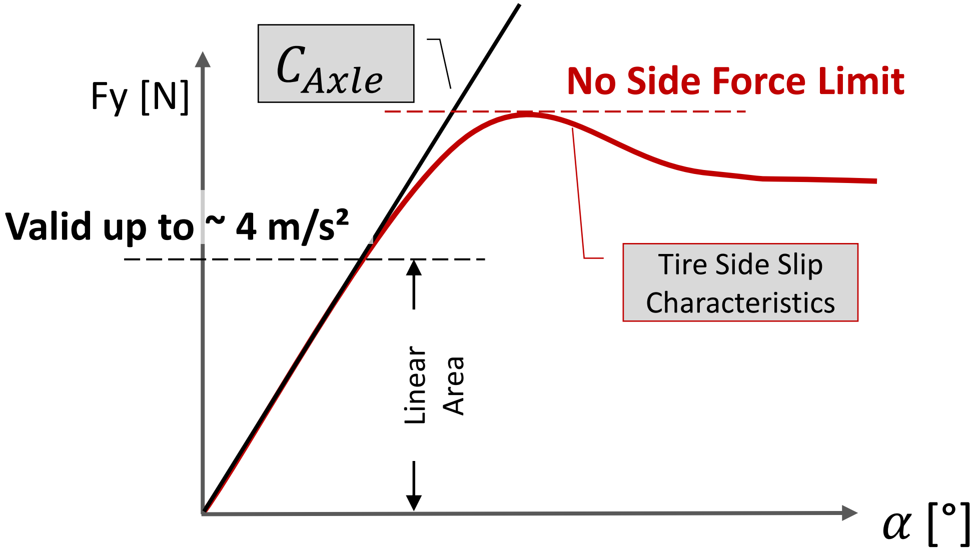
(1)
$$F_y = \boxed{F_{y,FA}} + \boxed{F_{y,RA}} = m * a_y = m * v * (\dot{\psi} - \dot{\beta})$$

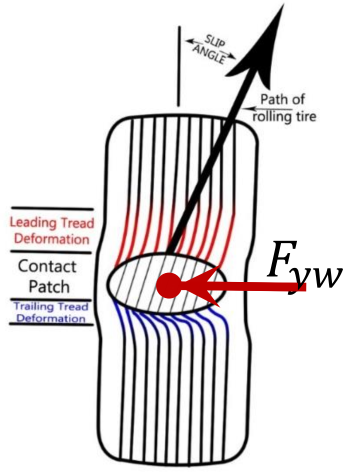
(4)
$$F_{y,FA} = c_{FA} * \boxed{\alpha_{FA}}$$

(5)
$$\alpha_{FA} = \delta + \beta - \frac{l_{FA} * \dot{\psi}}{v}$$

(4)
$$F_{y,RA} = c_{RA} * \boxed{\alpha_{RA}}$$

(5)
$$\alpha_{RA} = \beta + \frac{l_{RA} * \dot{\psi}}{v}$$





(5)

$$\alpha_{RA} = \beta + \boxed{\frac{l_{RA} * \dot{\psi}}{v}}$$

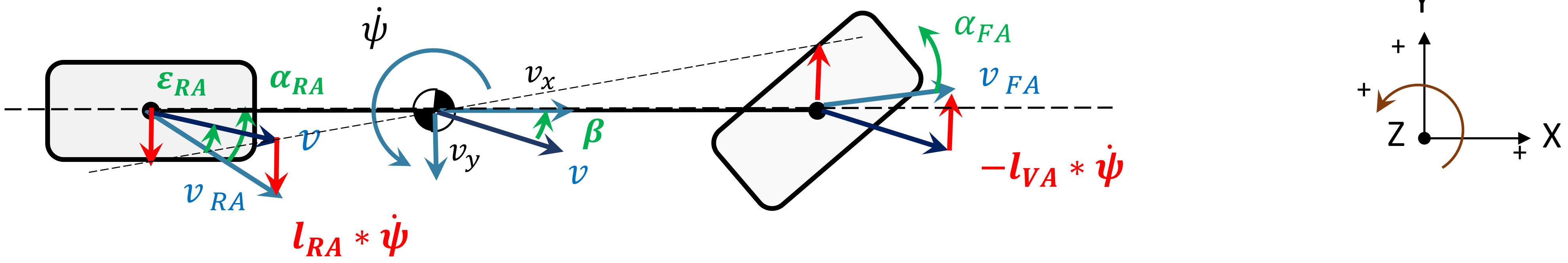
(5)

$$\alpha_{FA} = \delta + \beta \boxed{- \frac{l_{FA} * \dot{\psi}}{v}}$$

Small angle approximation

$$\epsilon_{RA} = \arctan \frac{l_{RA} * \dot{\psi}}{v} \sim \boxed{\frac{l_{RA} * \dot{\psi}}{v}}$$

$$\epsilon_{FA} = -\arctan \frac{l_{FA} * \dot{\psi}}{v} \sim \boxed{- \frac{l_{FA} * \dot{\psi}}{v}}$$



(9) $$c_{FA} * \boxed{\alpha_{FA}} = m * a_y * \frac{l_{RA}}{l}$$

(10) $$c_{RA} * \boxed{\alpha_{RA}} = m * a_y * \frac{l_{FA}}{l}$$

with 5

with 5

(11) $$c_{FA} \boxed{(\delta + \beta - \frac{l_{FA} * \dot{\psi}}{v})} = m * a_y * \frac{l_{RA}}{l}$$

(12) $$c_{RA} * \boxed{(\beta + \frac{l_{RA} * \dot{\psi}}{v})} = m * a_y * \frac{l_{FA}}{l}$$

(13) $$\beta = \frac{m}{l} * a_y * \frac{l_{RA}}{c_{FA}} + \frac{l_{FA} * \dot{\psi}}{v} - \delta$$

(14) $$\beta = \frac{m}{l} * a_y * \frac{l_{FA}}{c_{RA}} - \frac{l_{RA} * \dot{\psi}}{v}$$

(15) equalize

$$\delta = \frac{m}{l} * a_y * \frac{l_{RA}}{c_{FA}} - \frac{m}{l} * a_y * \frac{l_{VA}}{c_{RA}} + \frac{l_{FA} * \dot{\psi}}{v} + \frac{l_{RA} * \dot{\psi}}{v}$$

Basic vehicle dynamics calculation and vehicle models

(16) $$\delta = \boxed{\frac{m}{l} * a_y * \frac{l_{RA}}{c_{FA}} - \frac{m}{l} * a_y * \frac{l_{VA}}{c_{RA}}} + \boxed{\frac{l_{FA} * \dot{\psi}}{v} + \frac{l_{RA} * \dot{\psi}}{v}}$$

(17) $$\delta = \boxed{\frac{m}{l} * a_y * \left[\frac{l_{RA}}{c_{FA}} - \frac{l_{VA}}{c_{RA}}\right]} + \boxed{\boxed{\frac{\dot{\psi}}{v}} * \boxed{(l_{FA} + l_{RA})}}$$

$$\boxed{\frac{1}{r}} \qquad \boxed{l}$$

$$\boxed{\delta = \frac{l}{r} + \frac{m}{l} * a_y * \left[\frac{l_{RA}}{c_{FA}} - \frac{l_{FA}}{c_{RA}}\right]}$$

Steady-state cornering

(18) $$\delta = \frac{l}{r} + \frac{m}{l} * a_y * \left[\frac{l_{RA}}{c_{FA}} - \frac{l_{FA}}{c_{RA}}\right]$$

Ackermann

Self-Steering Behavior

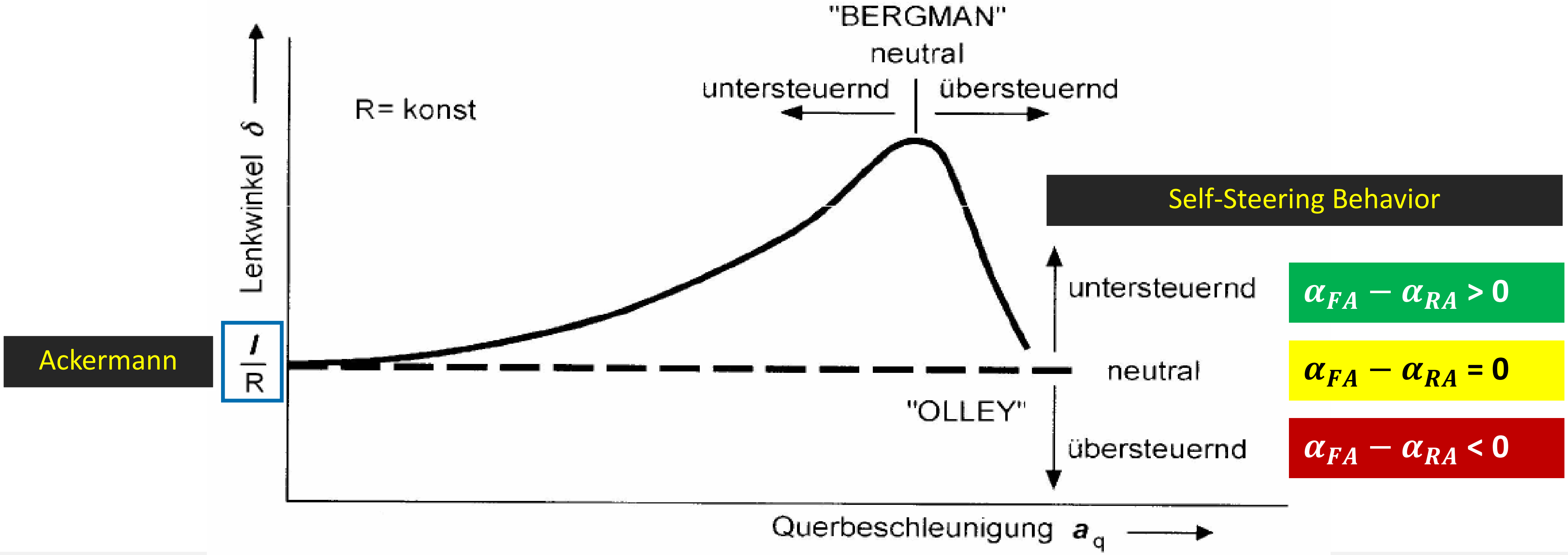
(19) $$\delta = \frac{l}{r} + \Delta\alpha \quad (\Delta\alpha = \alpha_{FA} - \alpha_{RA})$$

Understeer and oversteer definition

Ackermann

Self-Steering Behavior

$$\delta = \frac{l}{r} + \Delta\alpha \quad (\Delta\alpha = \alpha_{FA} - \alpha_{RA})$$



Definition of Eigenlenkgradient (Understeer Gradient)

(20) **Calculation of Eigenlenkgradient**

$$EG = \frac{1}{i_s} * \frac{d\delta_H}{da_y} - \frac{d\delta_A}{da_y}$$

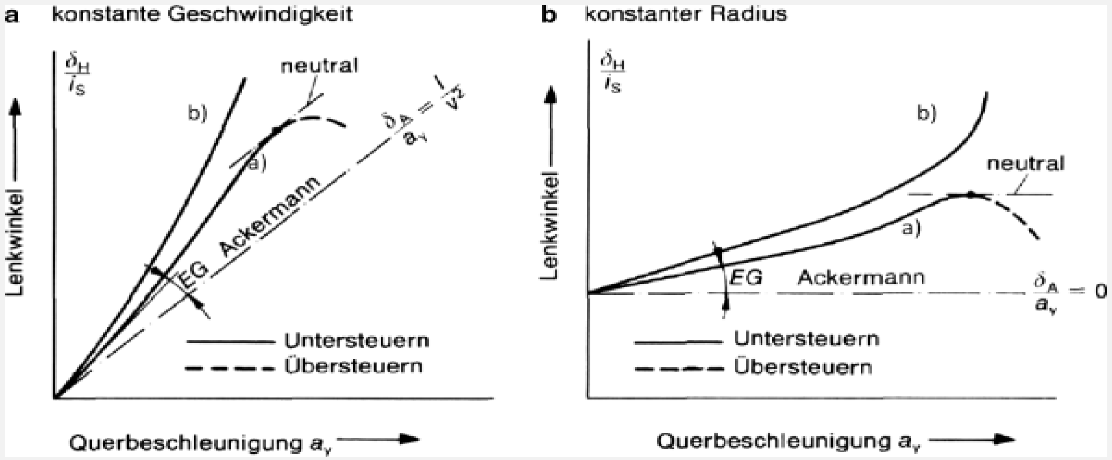
$= 0 \text{ if } R = const \text{ since } d\delta_A \approx \frac{l}{R}$

Calculation of **the EG** = Eigenlenkgradient (engl. understeer gradient)
$\boldsymbol{d\delta_H}$: Steering wheel angle
$\boldsymbol{d\delta_A}$: Ackermann angle

(21) **Calculation of specific Eigenlenkgradient**

$$EG_H = \frac{d\delta_H}{da_y} - \frac{d\delta_A}{da_y}$$

H = Hand at Steering Wheel



Definition of Eigenlenkgradient (Understeer Gradient)

Calculation of Eigenlenkgradient

(20)

$$EG = \frac{1}{i_s} * \frac{d\delta_H}{da_y} - \frac{d\delta_A}{da_y}$$

$= 0$ if $R = const$ since $d\delta_A \approx \frac{l}{R}$

Calculation of **the EG** = Eigenlenkgradient (engl. understeer gradient)
$\boldsymbol{d\delta_H}$: Steering wheel angle
$\boldsymbol{d\delta_A}$: Ackermann angle

Calculation of specific Eigenlenkgradient

(21)

$$EG_H = \frac{d\delta_H}{da_y} - \frac{d\delta_A}{da_y}$$

H = Hand at Steering Wheel

$EG = 0$: Neutral
$EG < 0$: Oversteer
$EG > 0$: Understeer

Characteristic Velocity and Critical Velocity

(22) **Calculation of Characteristic Velocity**

$$\left[\frac{\dot{\psi}}{\delta_H}\right]_{v_{char}} = \frac{1}{2} * \left[\frac{\dot{\psi}}{\delta_H}\right]_{EG=0}$$

if $\frac{\dot{\psi}}{\delta_H} > \left[\frac{\dot{\psi}}{\delta_H}\right]_{EG=0}$ and $R = const$

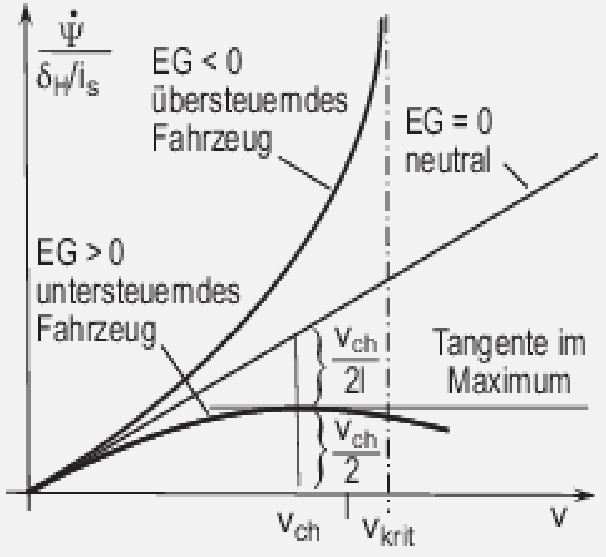
i.e. if vehicle shows steady-state understeer

(23) **Calculation of Critical Velocity**

$$\left[\frac{\dot{\psi}}{\delta_H}\right]_{v_{crit}} = \frac{1}{2} * \left[\frac{\dot{\psi}}{\delta_H}\right]_{EG=0}$$

if $\frac{\dot{\psi}}{\delta_H} < \left[\frac{\dot{\psi}}{\delta_H}\right]_{EG=0}$ and $R = const$

i.e. if vehicle shows steady-state oversteer



Practice Session: 15 min

Vehicle Dynamics in context of Advanced Driver Assistance Systems and Automated Driving

Hochschule Kempten

You have a vehicle with following data:

- m=1600 kg
- Wheel base = 2540 mm, Track width = 1420 mm,
- mFA = 880 kg, mRA = 720 kg,
- Steering ratio = 1:15,
- Yaw interia moment = 2800 kg m²,
- Side slip stiffness front/rear = 3000 N/°

1. Please calculate the center of gravity (GoG) in X-direction.

2. Please calculate Ackermann angle for a circle of R=100 m.

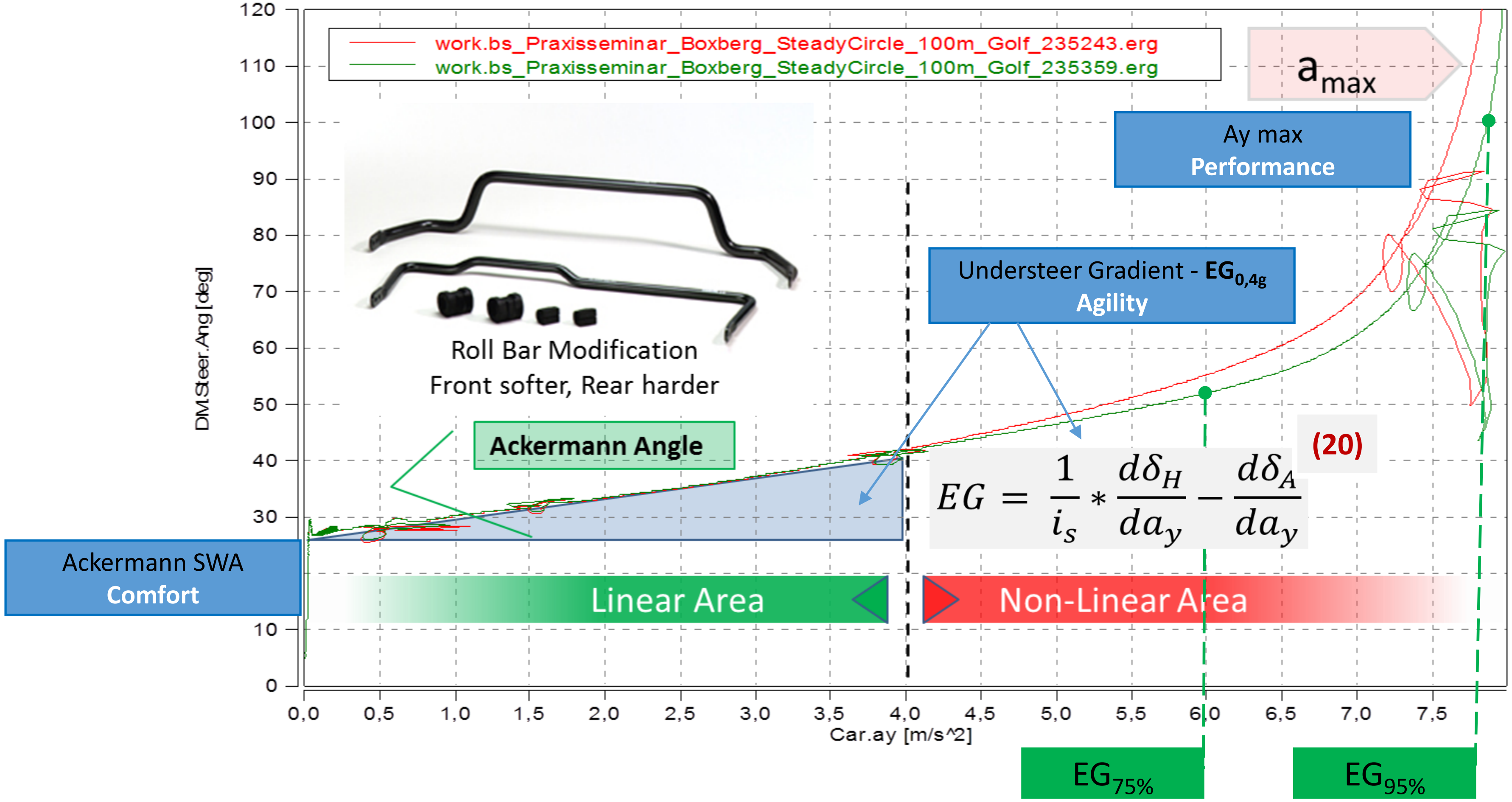
3. Calculate the Eigenlenkgradient (understeer gradient) between 0 – 4 m/s².

Practice Session: 15 min

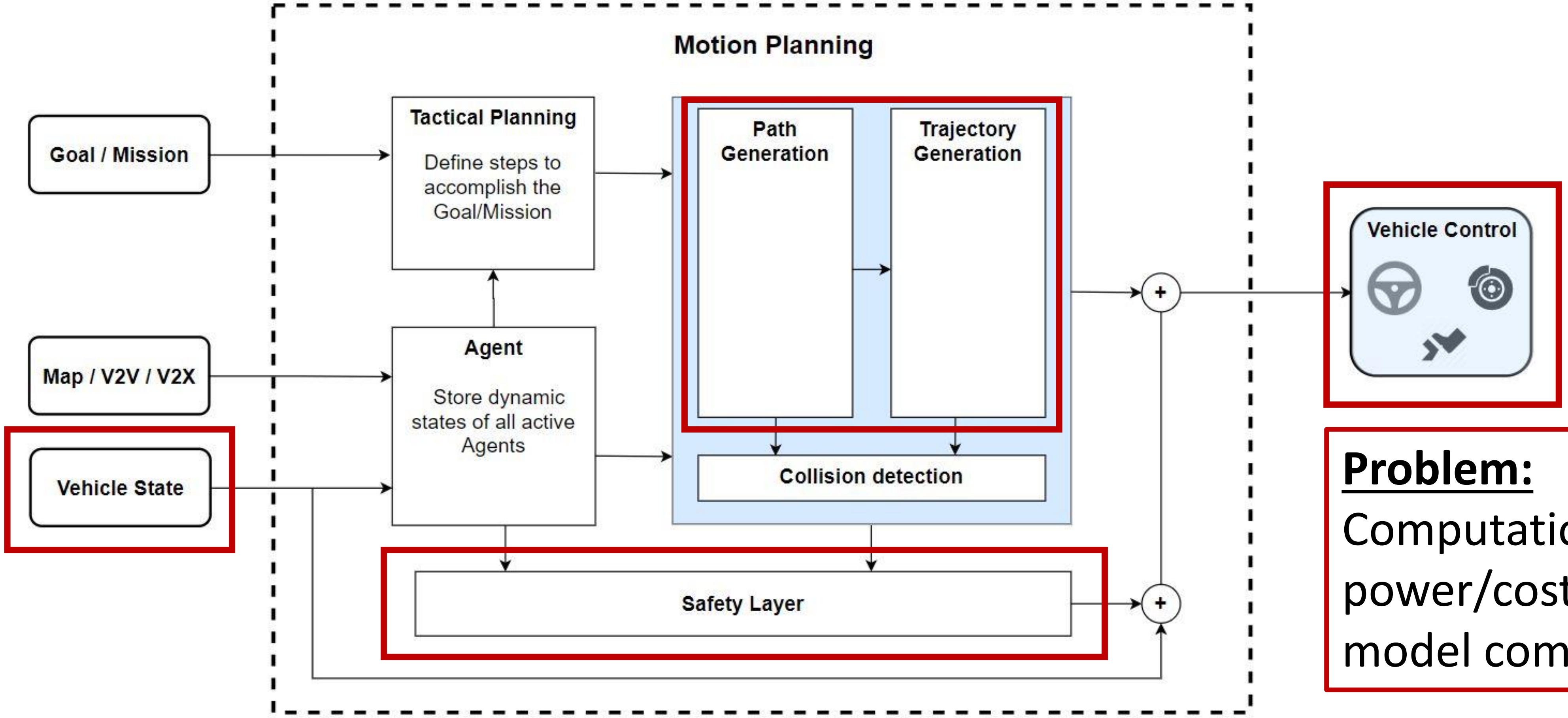
5. How big is the steering angle for the driver at 4 m/s^2?

6. Which measures do you recommend to reduce the understeer gradient to appr. 50%? Please describe 3

Model behavior in steady-state cornering

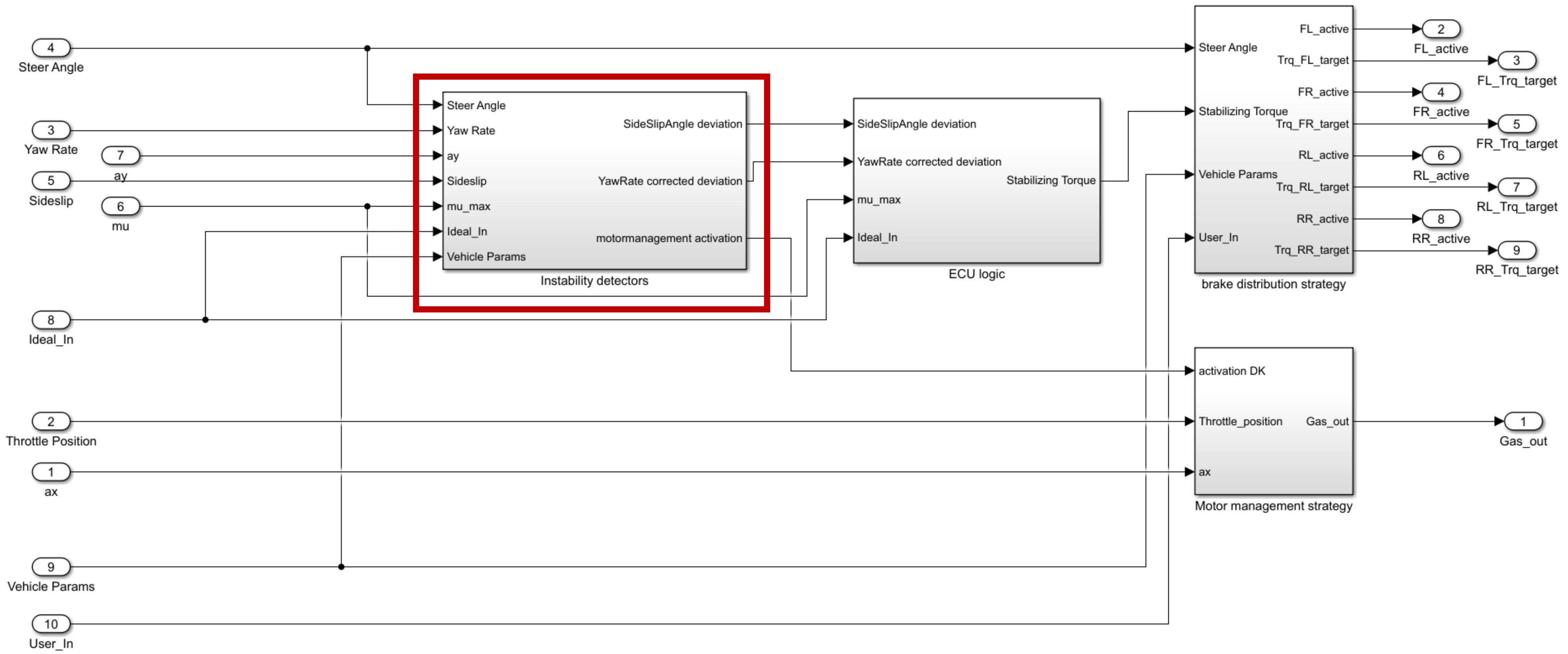


Use-Cases for a Single-track model – Model-based motion planning

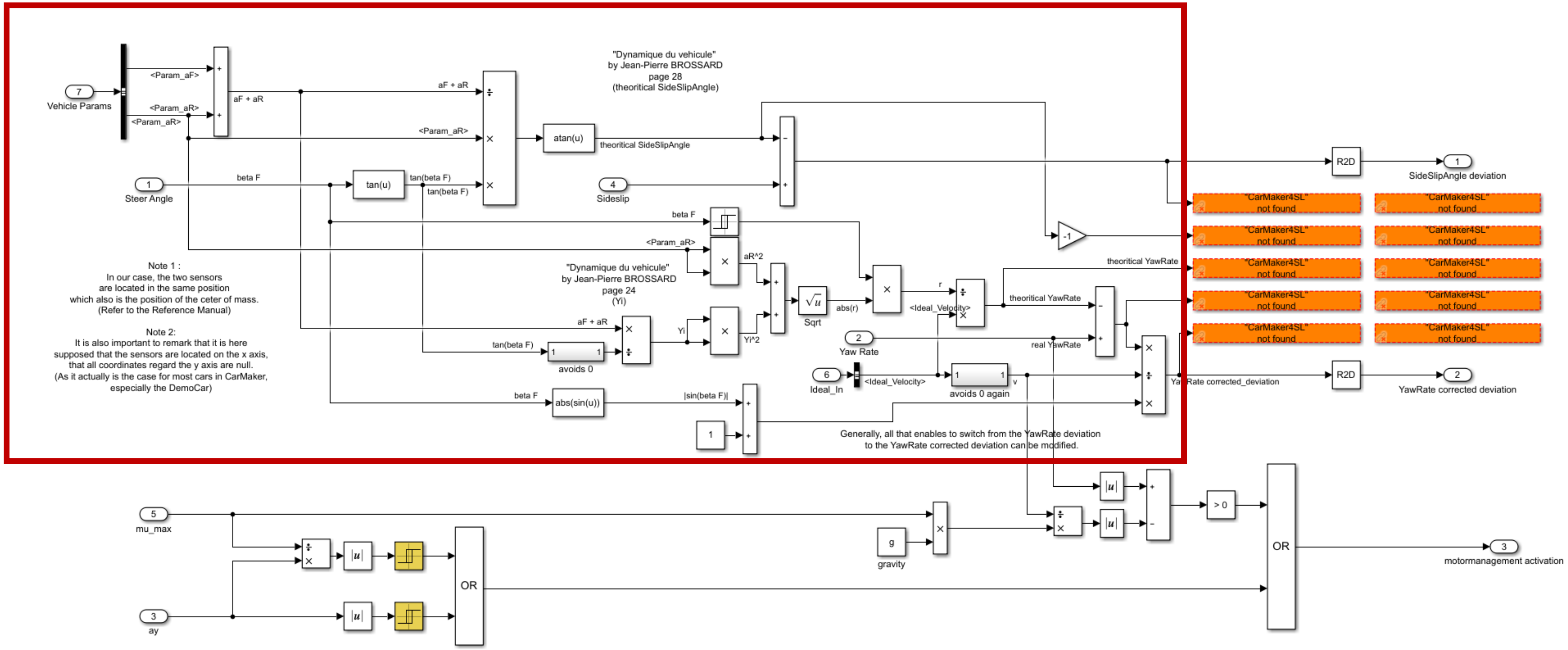


Problem:
Computational power/cost limits model complexity

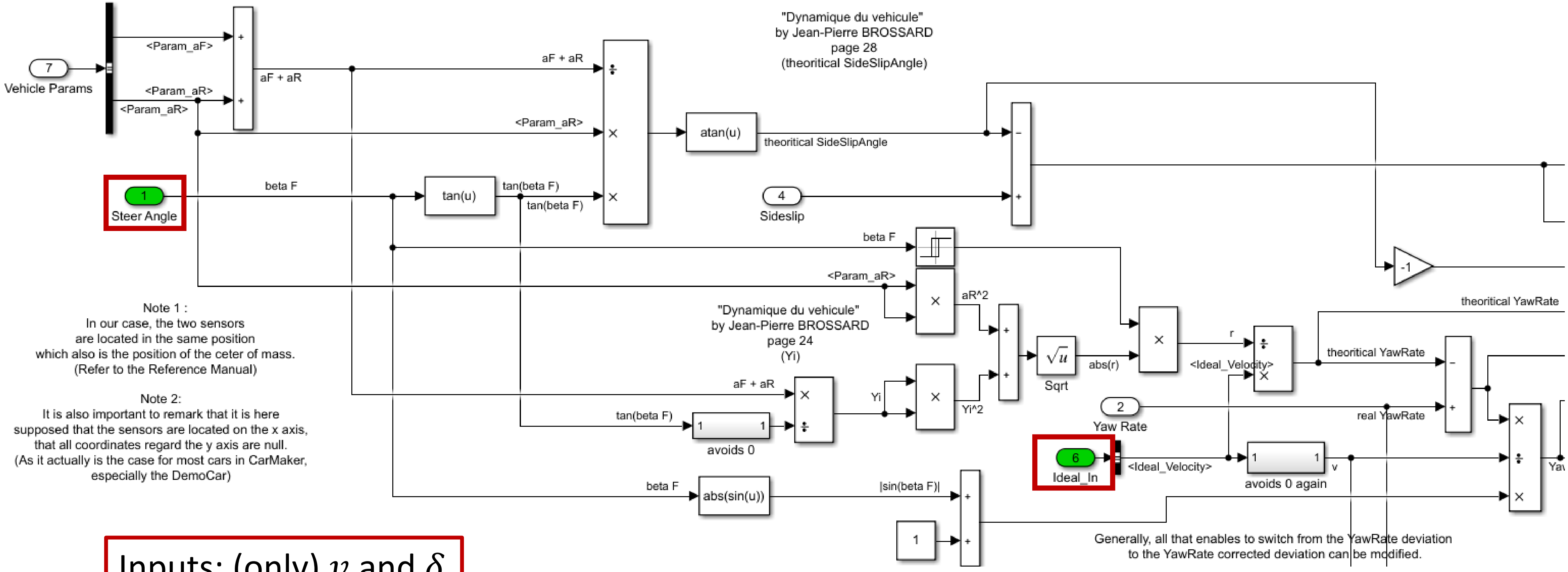
Use-Cases for a Single-track model – Model-based control



Use-Cases for a Single-track model – Model-based control

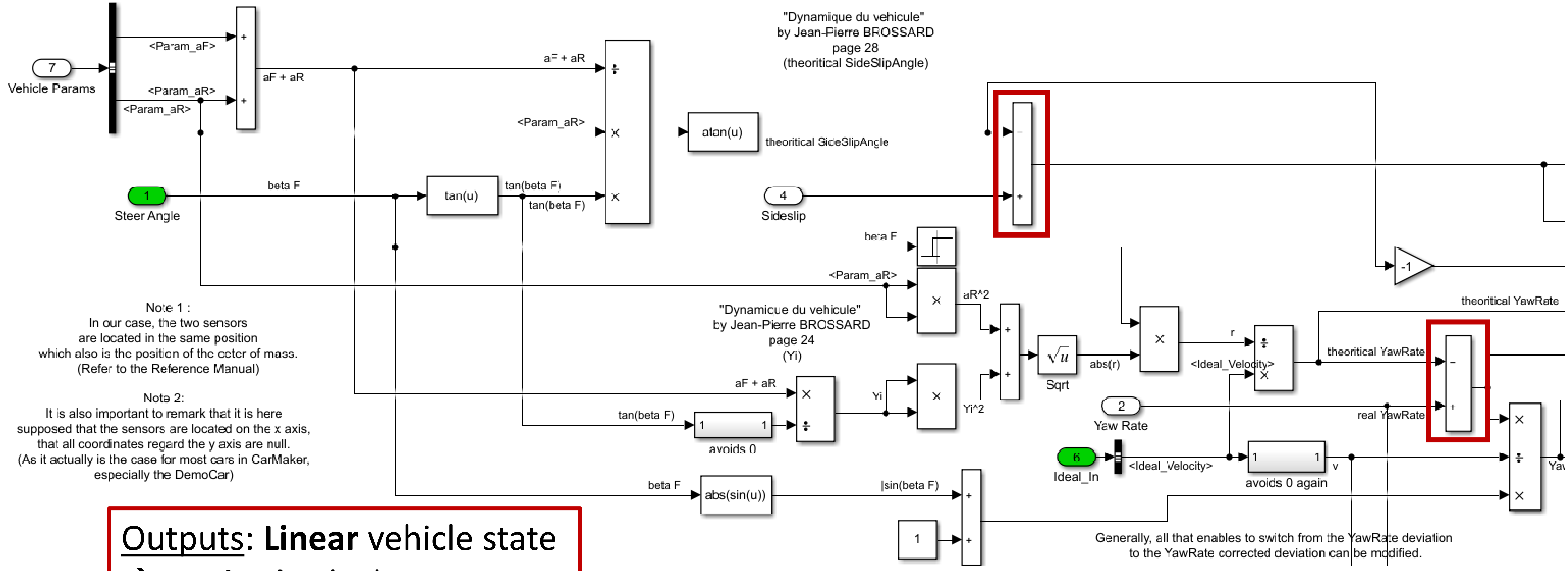


Use-Cases for a Single-track model – Model-based control



Inputs: (only) v and δ

Use-Cases for a Single-track model – Model-based control



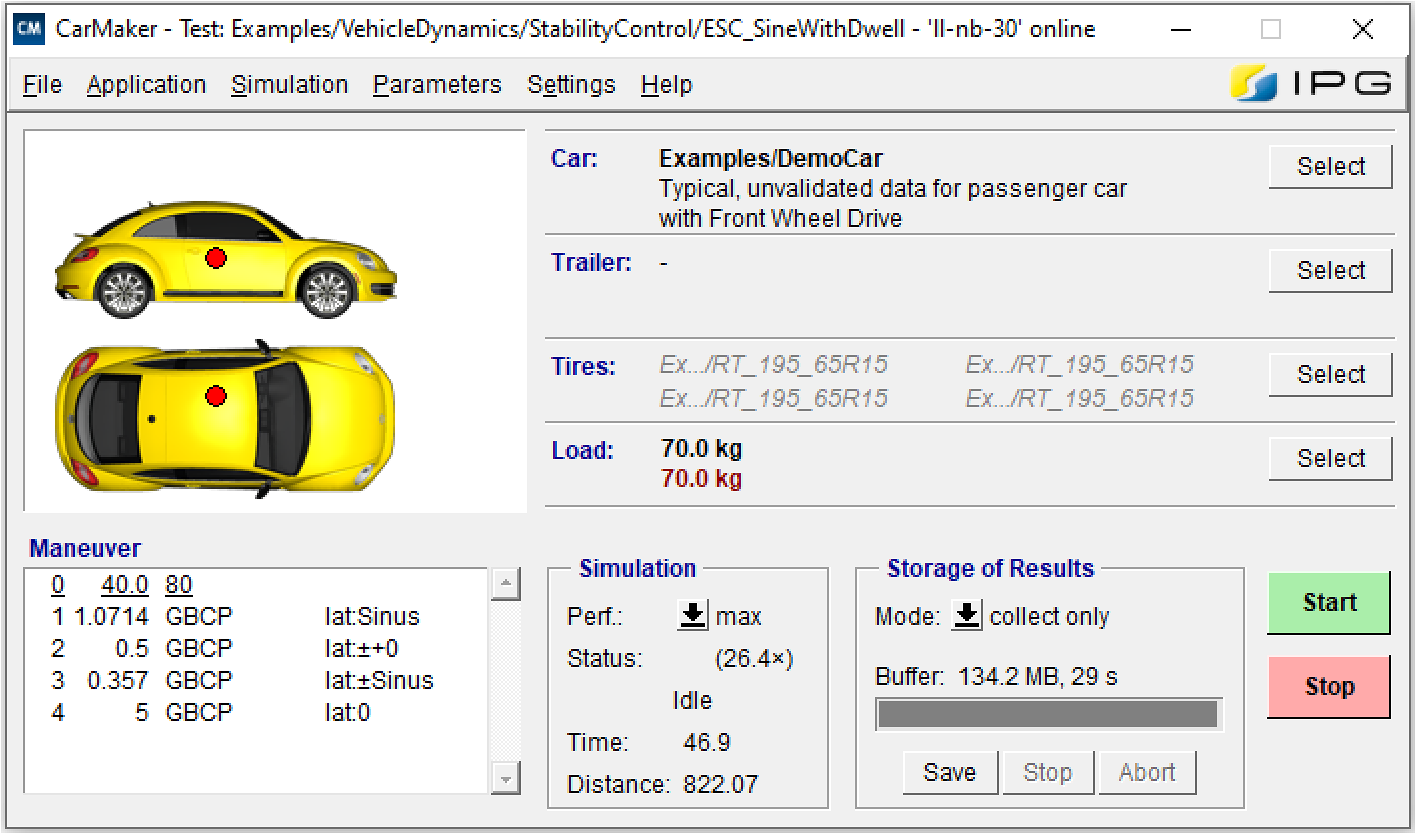
<u>Outputs</u>: **Linear** vehicle state

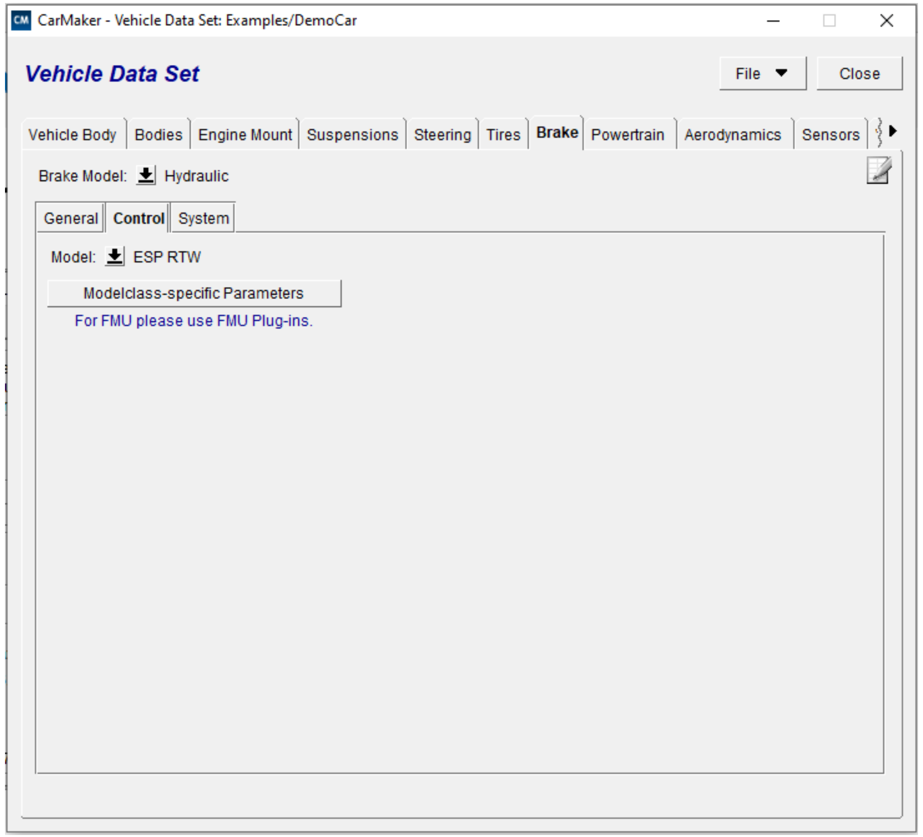
→ **Desired** vehicle response

drive.livinglab

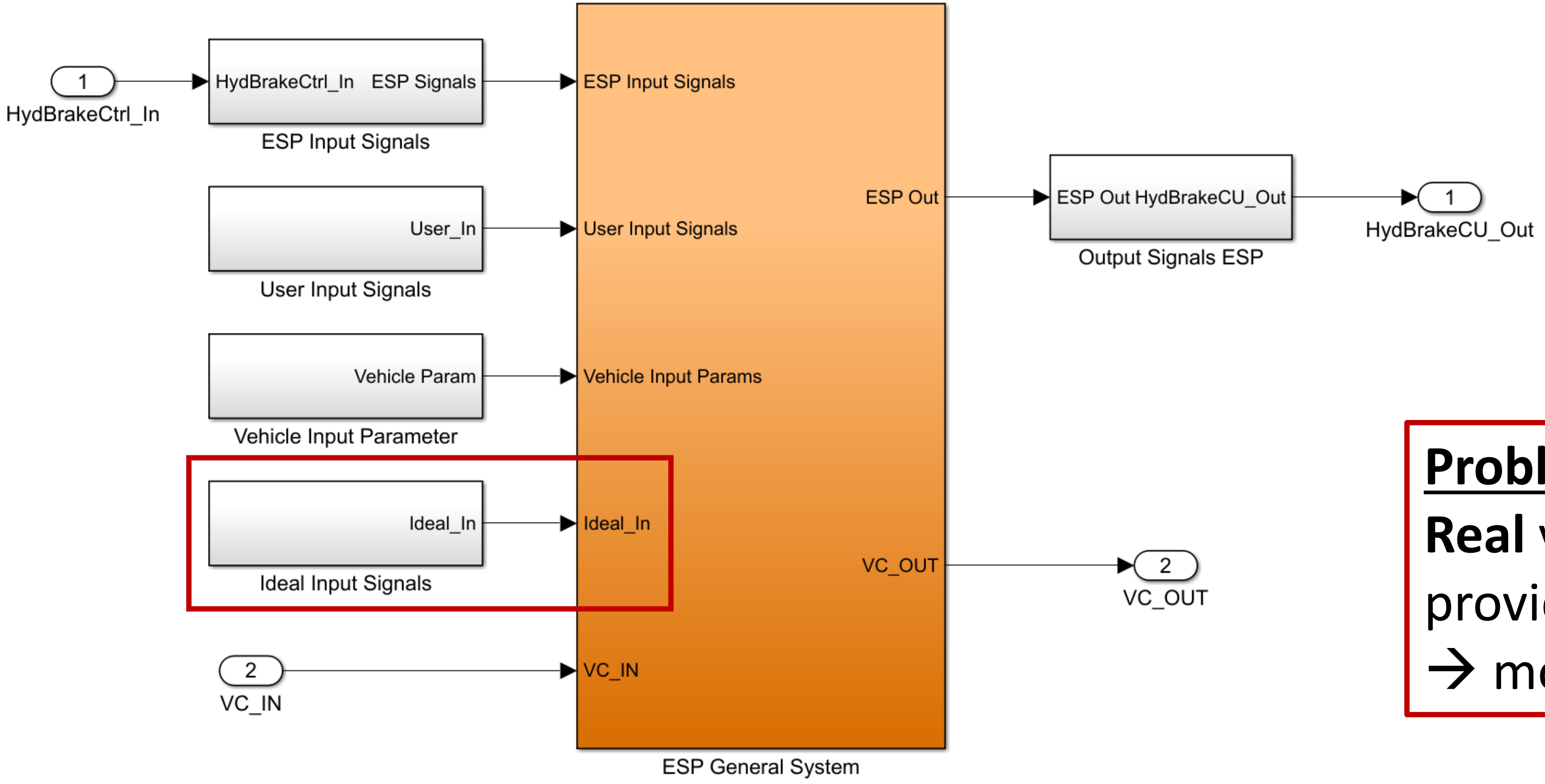
Kempten University of Applied Sciences

Model-based control – CarMaker example





Use-Cases for a Single-track model – Vehicle state estimation



Problem:

Real vehicles don't provide **ideal** in-/outputs

→ model-based estimate

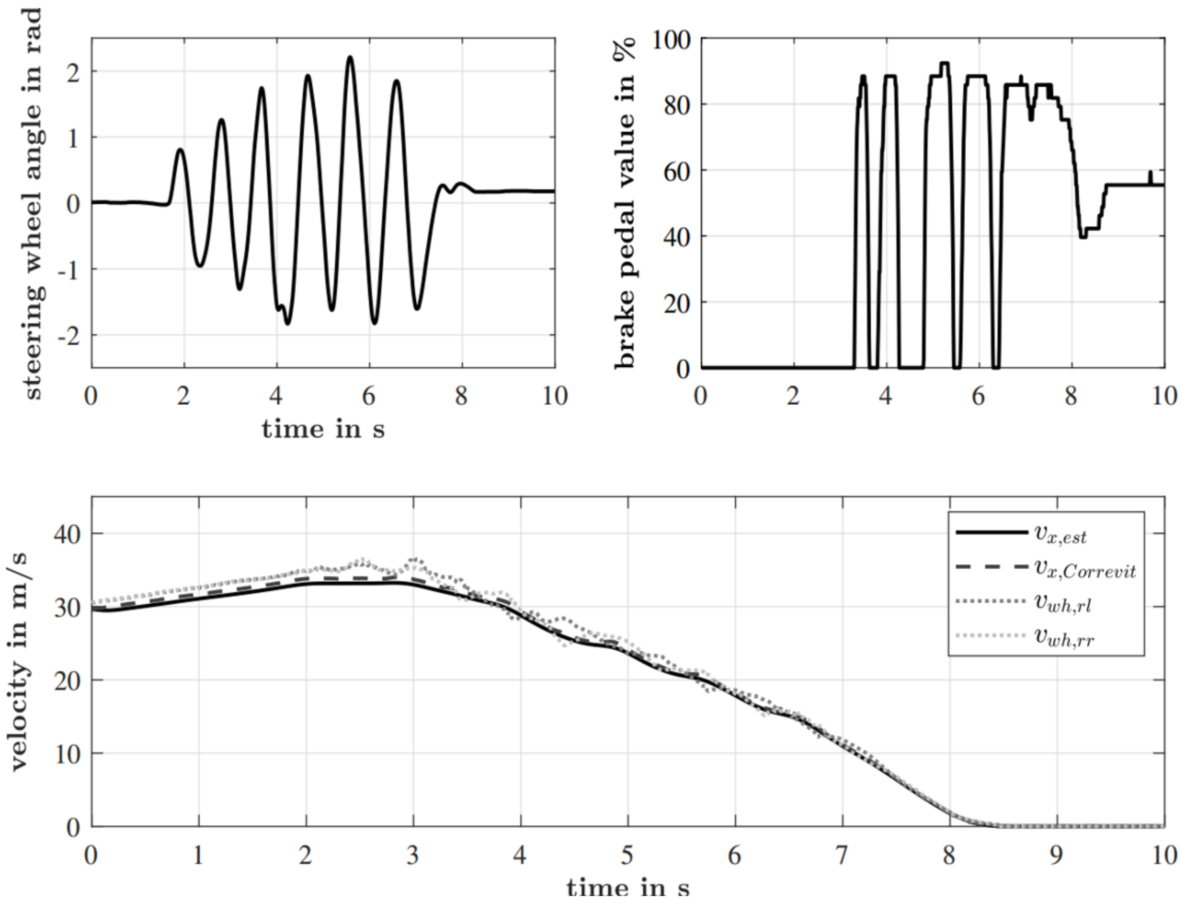
Use-Cases for a Single-track model – Vehicle state estimation

<u>Estimation of non-measurable quantities</u>

- **Longitudinal velocity** under slip
- **Lateral velocity**
- Tire **forces**
- Available **friction coefficient** (tire potential)
- Road bank angle
- …

<u>Prediction of vehicle state</u>

- Motion planning
- Advanced control



What can we improve? – Extensions of the Single-Track model

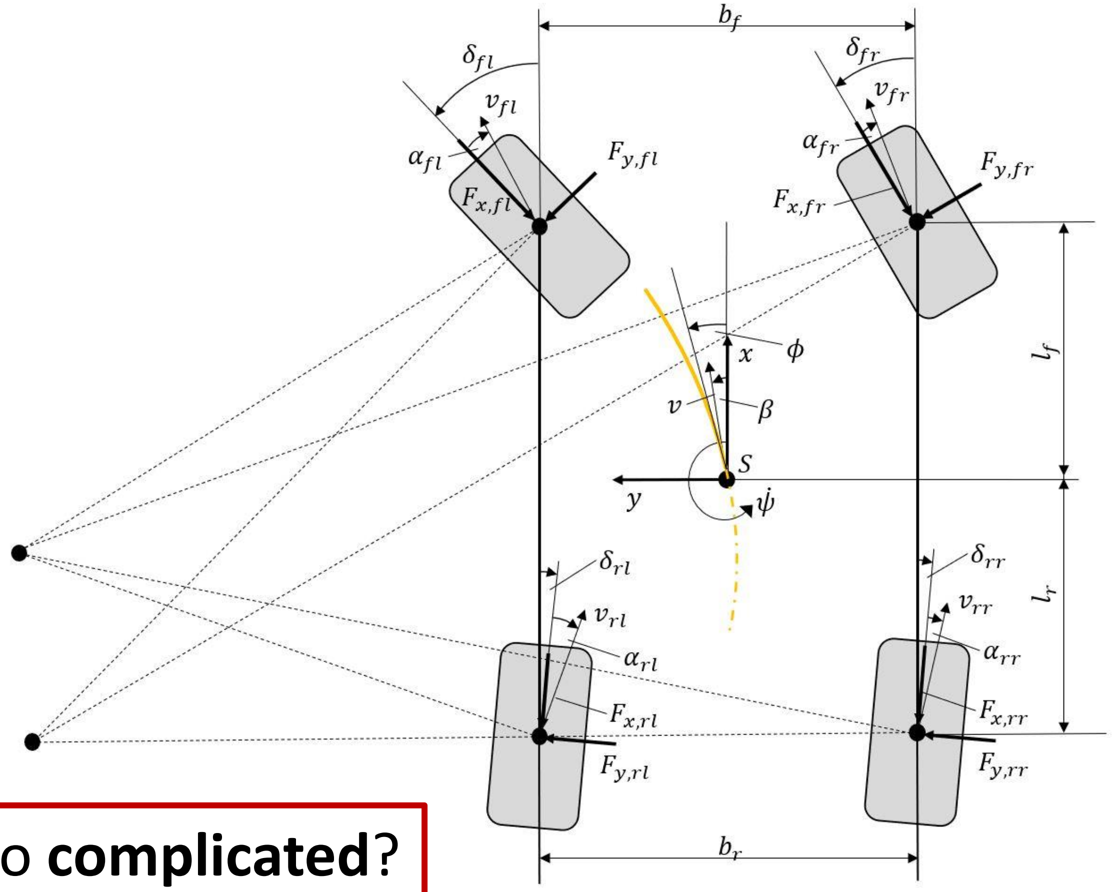
<u>Two-track model (**6 DOF**)</u>

- Consider **roll, heave** and **pitch** motion
  → **CoG height** is now relevant
  → Four wheels with **dynamic wheel load transfer**

<u>Non-linear tire modeling</u>

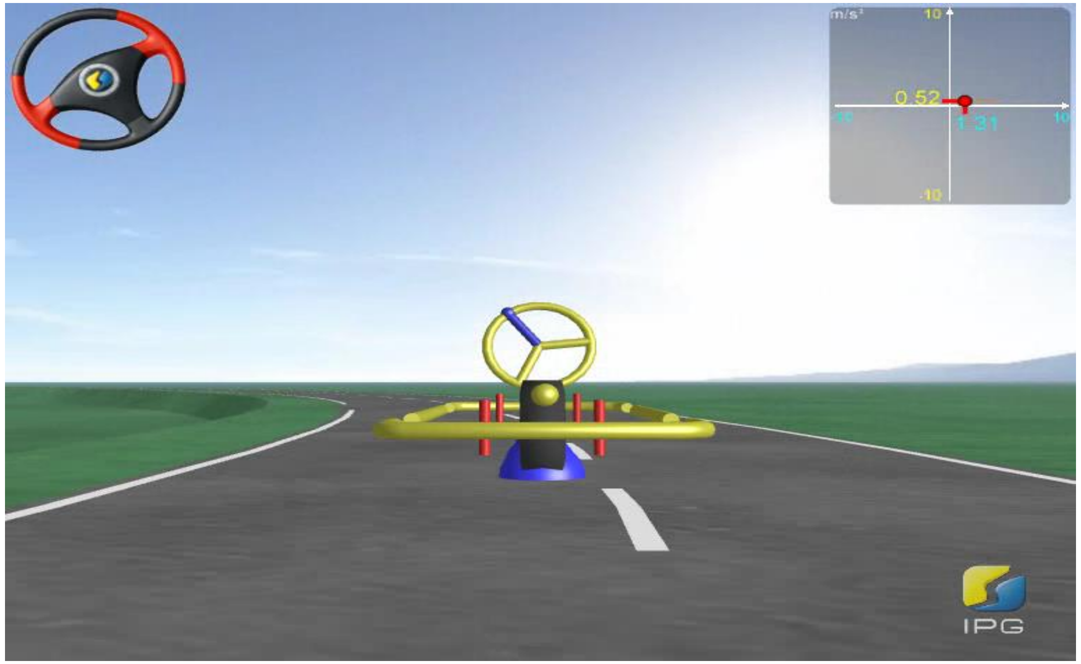
- Saturation through long./lat. force limits (tire potential)

- Steering angle on both axles
- Slip-angle contribution of resulting axle stiffness
- Transient vertical dynamics
- Combined slip modeling
- …

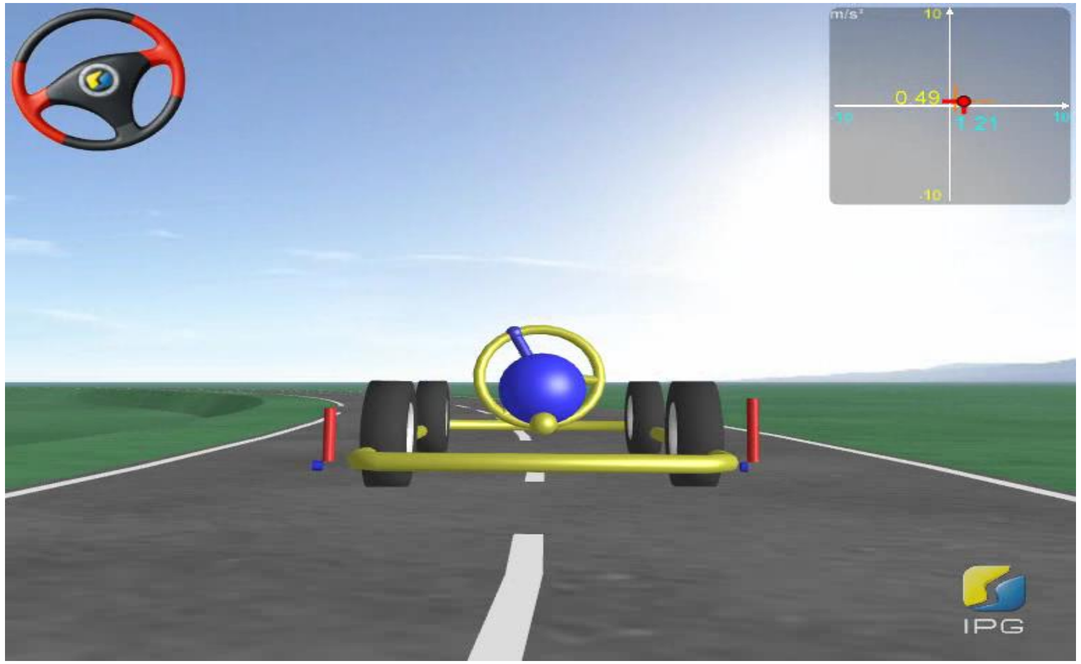


→ How **complex** is too **complicated**?

Comparison Single Track Model with 3D-MBS Model in circular driving



Single Track Model
(linear)



Reduced MBS 3D-Model
with empirical parts

Validity of a single-track model: CarMaker exercise