



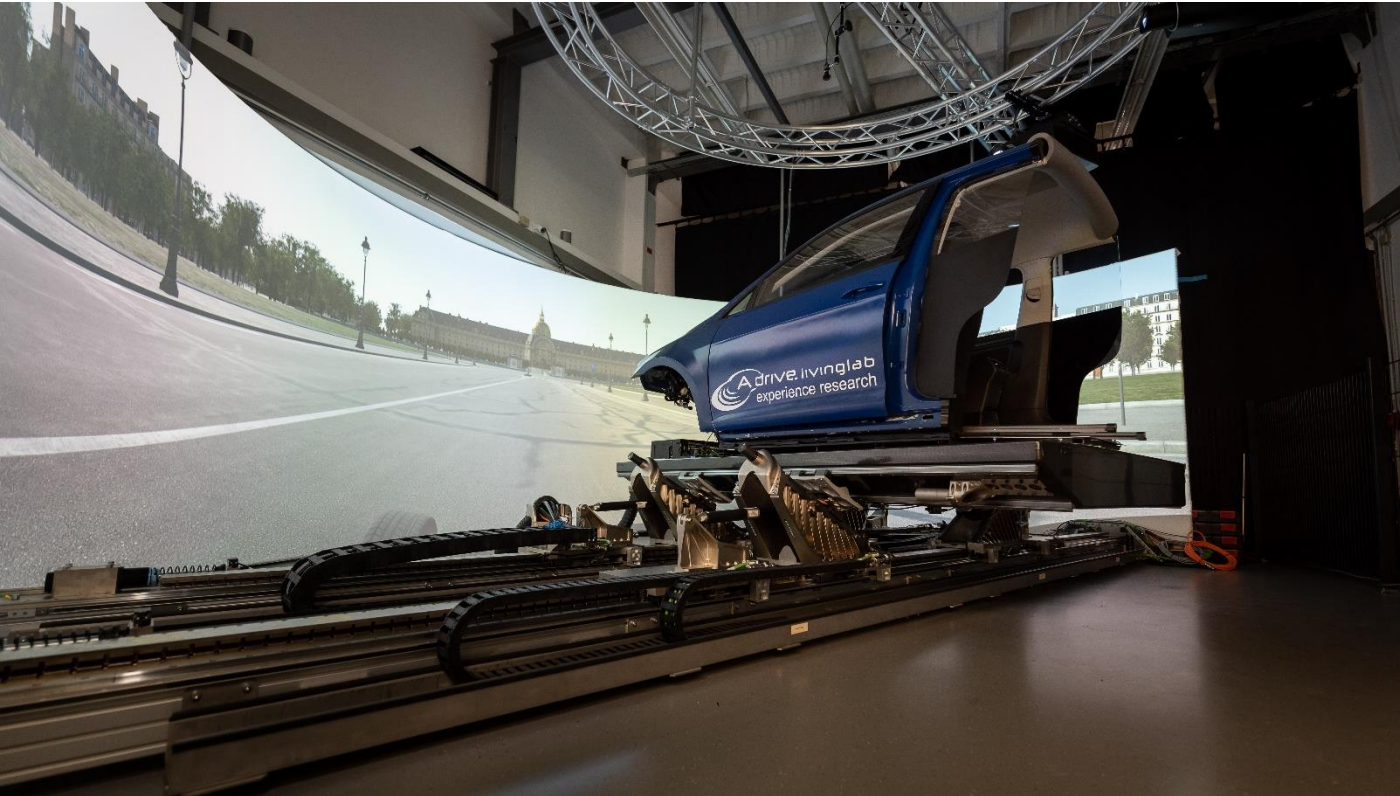
Basic vehicle dynamics calculation and vehicle models



<https://moodle2.hs-kempten.de/moodle/course/view.php?id=2914>

Nr.	Datum	Inhalt	Ort	Von Wem
1	19.03.	Introduction Vehicle dynamics <ul style="list-style-type: none">• Principles• Vehicle attributes and their target conflicts• Subjective & objective evaluation• Measurement methods• Introduction and Installation CarMaker and MXeval• Virtual Test Driving (VTD) CarMaker Quick Start Guide	Zoom	Schick
2	26.03	Practical Training Vehicle Dynamics Simulation and Evaluation <ul style="list-style-type: none">• Issue of PSA	Zoom	Schick Tarne
3	09.04.	Test and evaluation methods for vehicle attributes with practical simulation <ul style="list-style-type: none">• Steady State Behavior• Transient Behavior	I305 Zoom	Schick
4	16.04.	Chassis components and functions with practical simulation <ul style="list-style-type: none">• Tire & Wheels• Kinematic & Compliance• Steering Systems	I305 Zoom	Schick
5	30.04.	Basic vehicle dynamics calculation and vehicle models <ul style="list-style-type: none">• Introduction single track model• Characteristic and critical speed• Wheel load distribution calculation• Practical calculation exercise• Simulation exercise & comparison	I305 Zoom	Böhle

Basic vehicle dynamics calculation and vehicle models



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Agenda

- Basics
 - Forces, moments, masses
 - Equations of motion
 - Recap: Tires
- Basic calculations – dynamic
 - Self-steering behaviour
 - Practice session
- Validity and limitations
- Applications

Recommended literature

- Mitschke, Manfred, and Henning Wallentowitz. "Dynamik der Kraftfahrzeuge. 5., überarb. u. erg. Auflage." (2014).
 - Chapter 20: Lineares Einspurmodell, objektive Kenngrößen, Subjektivurteile
 - Chapter 21: Kreisfahrt bei konstanter Fahrgeschwindigkeit
- Ersoy, Metin, and Stefan Gies, eds. *Fahrwerkhandbuch: Grundlagen–Fahrdynamik–Fahrverhalten–Komponenten–Elektronische Systeme–Fahrerassistenz–Autonomes Fahren–Perspektiven*. Springer-Verlag, 2017.
 - Chapter 2: Fahrdynamik

Basic vehicle dynamics calculation and vehicle models

What is a model?



A simplified representation of the reality.

What is a vehicle dynamics model?



A tool to calculate the dynamic motion of ground vehicles for engineering tasks.

Where do we need vehicle dynamics models?



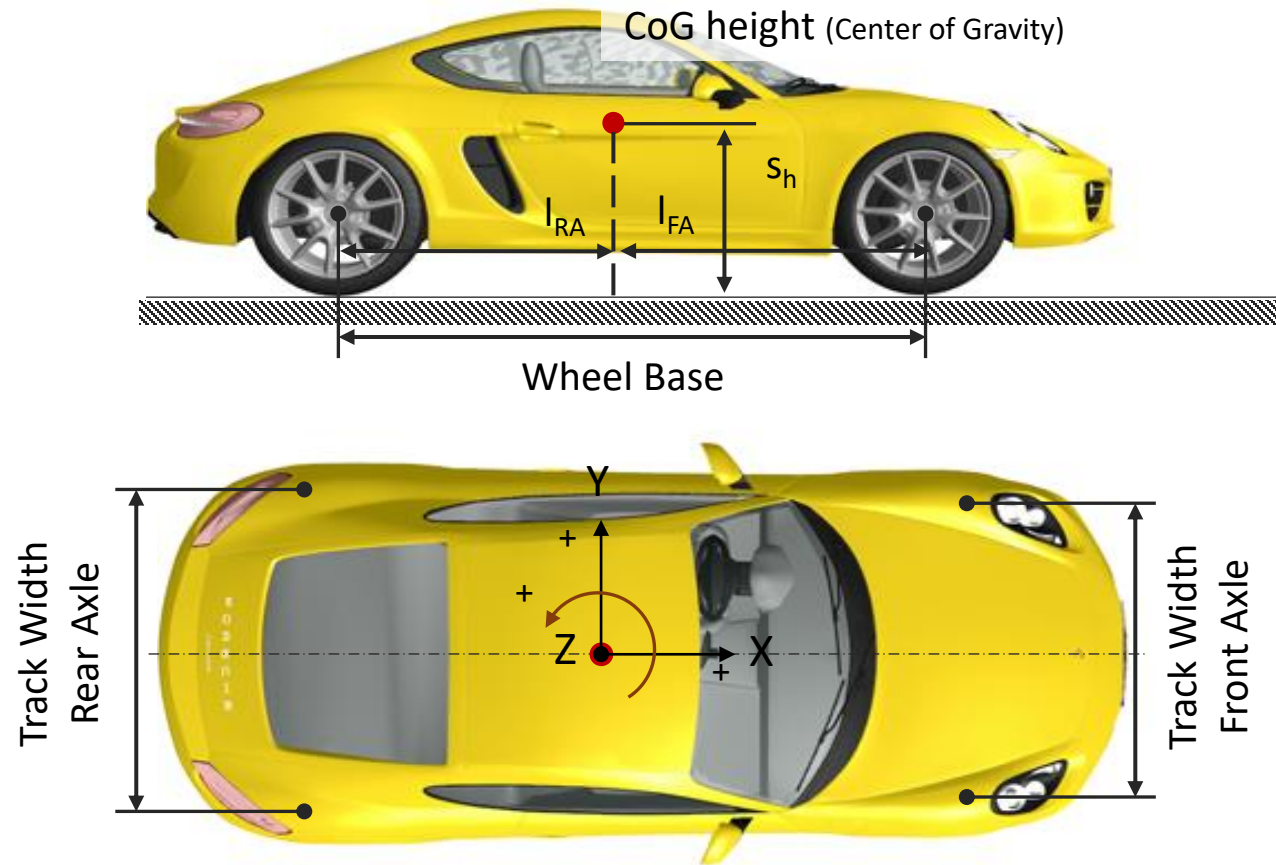
Simulation, model based testing and model based control methods.

When is a vehicle dynamics model valid?

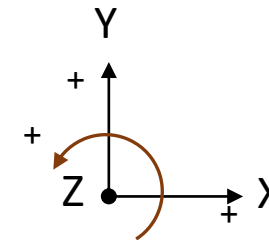
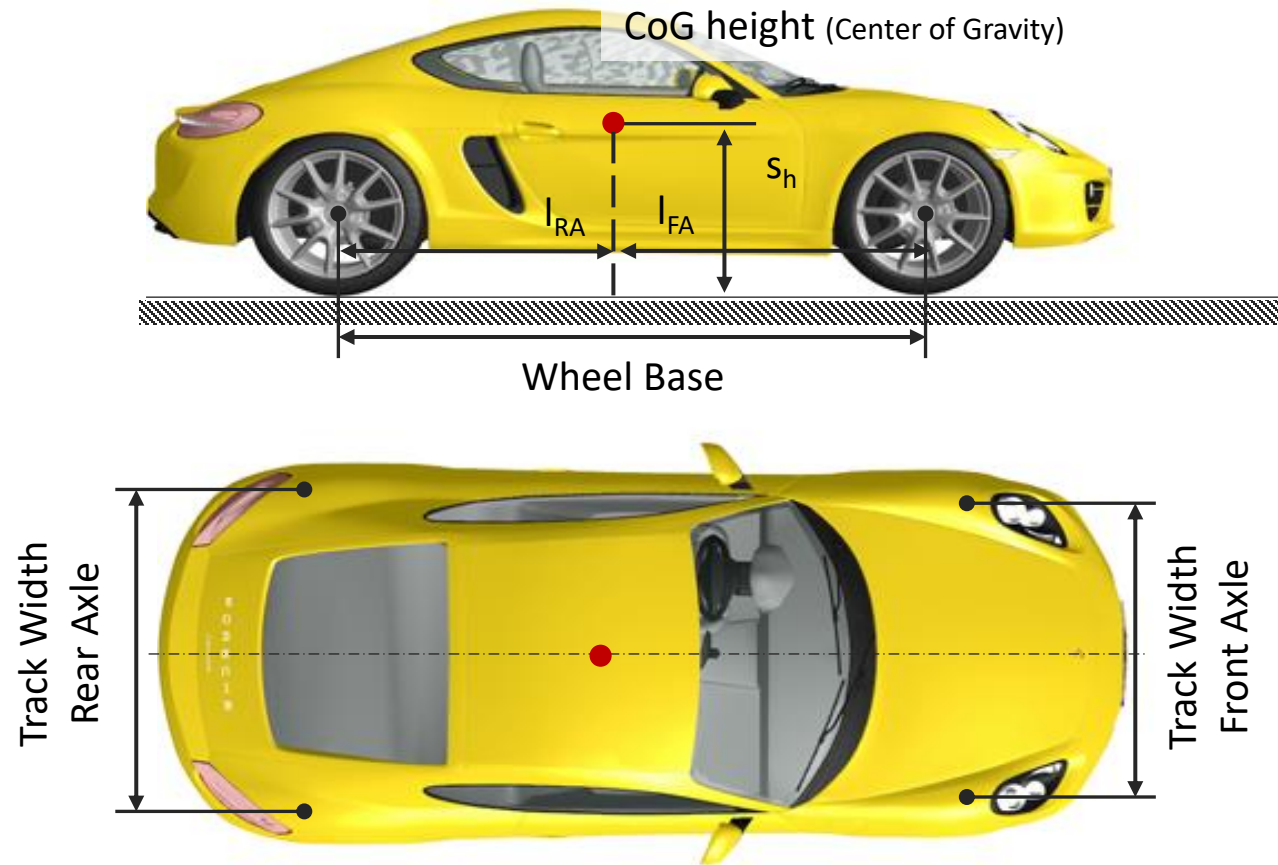


Good and accurate enough for the application purpose?

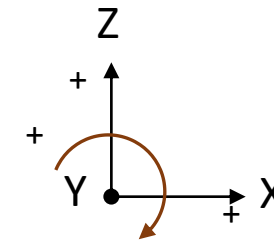
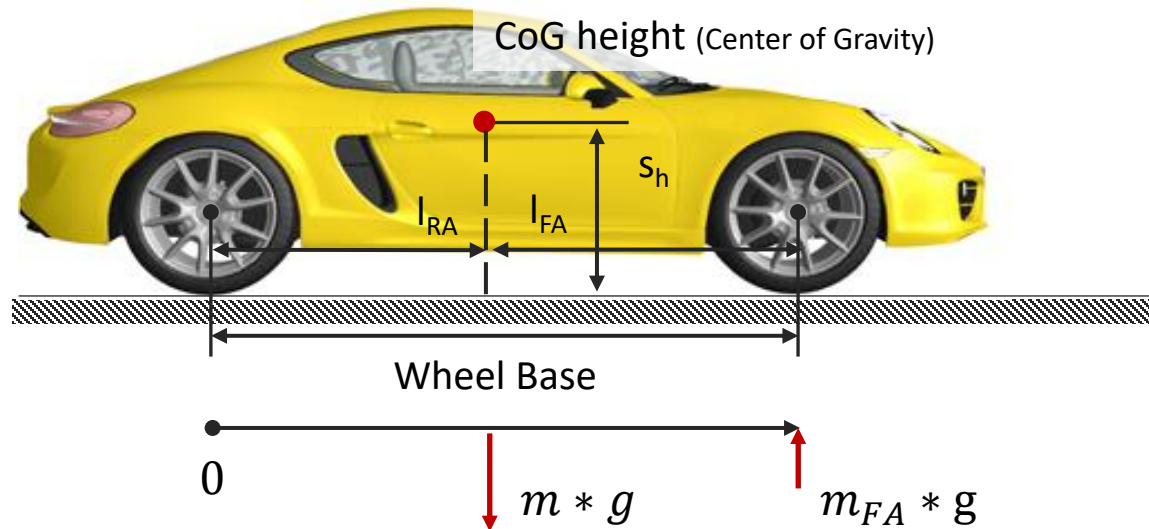
Basic chassis dimensions



Basic chassis dimensions



Calculation of CoG – Center of Gravity



$$\sum F_z = 0$$

$$m * g * l_{RA} = m_{FA} * g * l \quad l_{FA} = l - l_{RA}$$

- Vehicle weight (with driver) = 1.970 kg
- m_{FA} (front axle) = 1100 kg
- m_{RA} (rear axle) = 870 kg
- Wheel base = 2.807 m
- Center of gravity = 0,65 m

Calculation of CoG – Center of Gravity

$$m * \cancel{g} * l_{RA} = m_{FA} * \cancel{g} * l$$



$$l_{RA} = \frac{m_{FA} * l}{m} = \frac{1100 \text{ kg} * 2,807 \text{ m}}{1970 \text{ kg}} = 1,567 \text{ m}$$

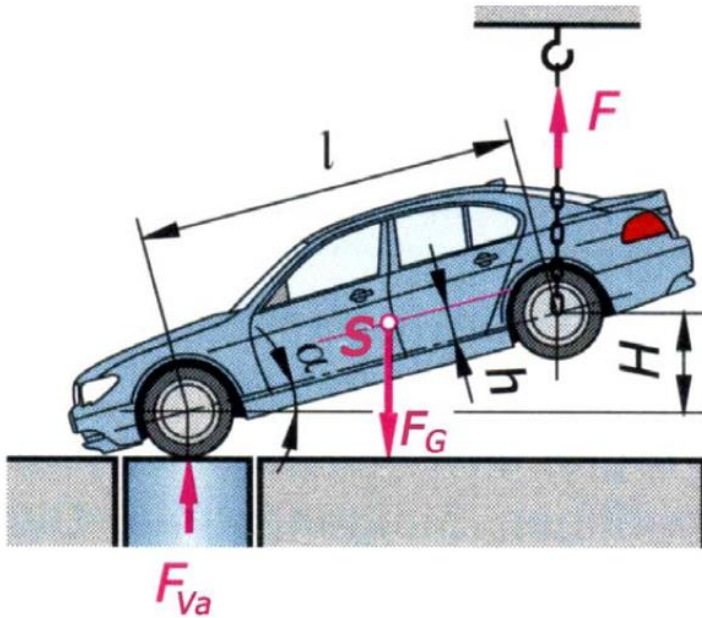
$$l_{FA} = l - l_{RA}$$



$$l_{FA} = l - l_{RA} = 2,807 \text{ m} - 1,567 \text{ m} = 1,24 \text{ m}$$

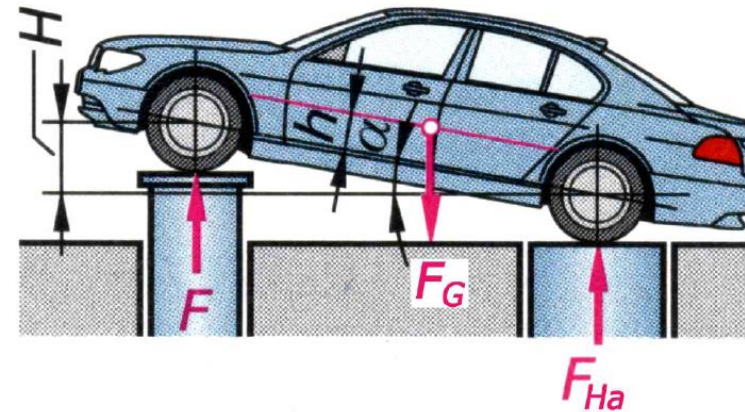
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- Wheel base = 2.807 mm,
- Center of gravity = 0,65 m

Calculation of CoG – Center of Gravity



[1]

$$h = l * \frac{F_{FA} - F}{F_G * \tan \alpha}$$



$$h = l * \frac{F_{HA} - F}{F_G * \tan \alpha}$$

Calculation of CoG – Center of Gravity

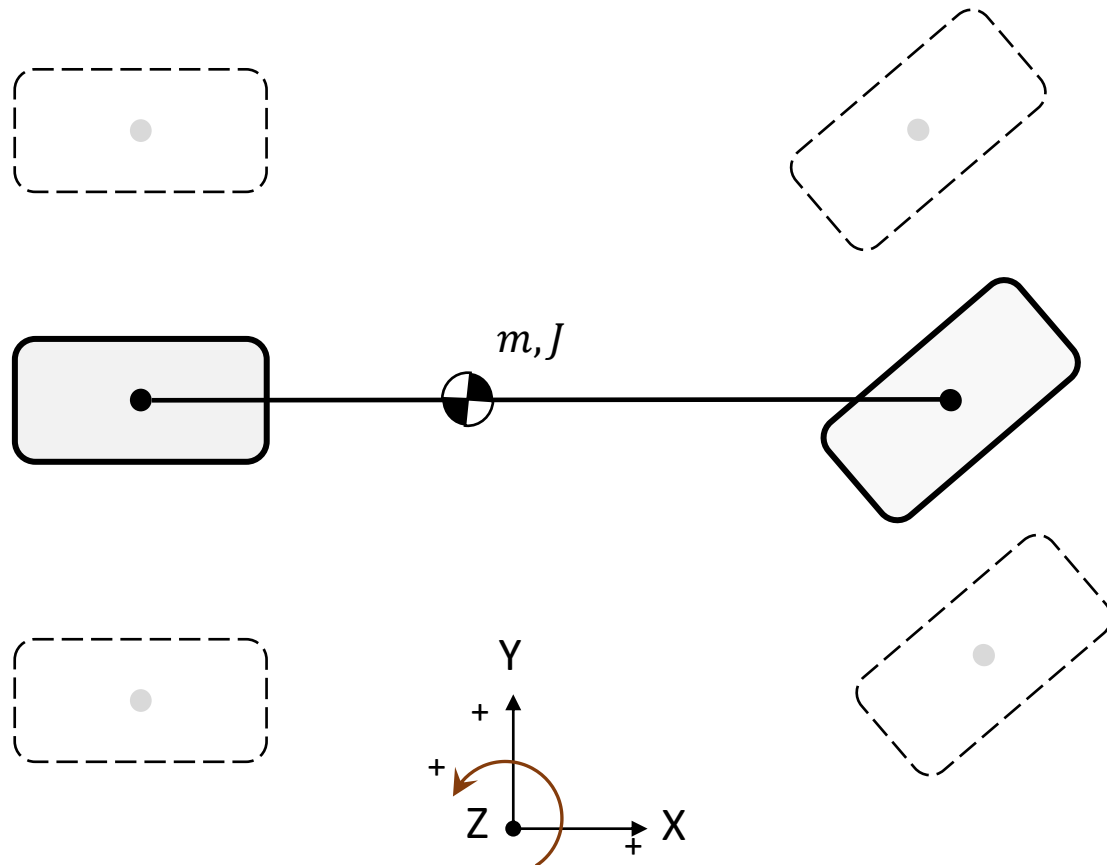


[2]



[3]

Simplification of the vehicle model: “Single Track Model” Theory - 3 DOF



Approach

- **Wheels are lumped into single track**
→ Tire side slip & axle stiffness are combined per axle
- Rigid body with CoG in-plane (on-track)
- Only horizontal movement
→ No roll, pitch & vertical motion
- Steering angle only at the front axle

3 Degrees of Freedom (DOF)

- Longitudinal
- Lateral
- Yaw (rotation around Z)

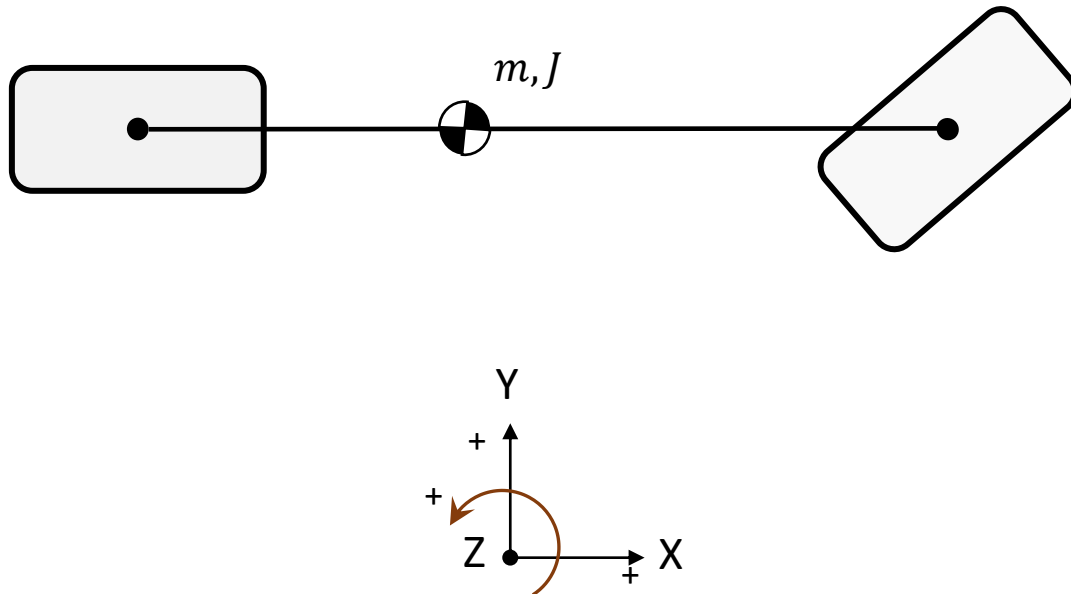
Simplification of the vehicle model: “Single Track Model” Theory - 3 DOF

Approach

- **Wheels are lumped into single track**
→ Tire side slip & axle stiffness are combined per axle
- Equations of motion based on:
 - Geometrical equation
 - Equilibrium of forces & moments
 - Transversal system stiffness (Tire / Axle)

3 Degrees of Freedom (DOF)

- Longitudinal
- Lateral
- Yaw (rotation around Z)



Notations

m	vehicle mass	δ	steering angle (at road wheel)
F_x, F_y	forces	δ_H	steering wheel angle (SWA, at hand wheel)
F_{x_w}, F_{y_w}	wheel forces	i_s	steering ratio
v	vehicle speed	ψ	yaw angle
v_x, v_y	longitudinal / lateral vehicle velocity	$\dot{\psi}$	yaw angle speed
a_x, a_y	longitudinal / lateral vehicle acceleration	R, r	course radius
v_{FA}, v_{RA}	velocity front / rear axle	FA, RA	Index front axle, rear axle
l	wheelbase	$stat$	index for stationary
l_{FA}, l_{RA}	front / rear axle distance to center of gravity	CoG	center of gravity
α_{FA}, α_{RA}	slip angle front / rear axle	EG	understeer gradient (Eigenlenkgradient)
c_{FA}, c_{RA}	cornering stiffness front / rear axle		
β	side slip angle		
$\dot{\beta}$	side slip angle velocity		

Basic vehicle dynamics calculation and vehicle models

(1)

Lateral Motion

$$F_y = F_{y,FA} + F_{y,RA} = m * a_y = m * v * (\dot{\psi} - \dot{\beta})$$

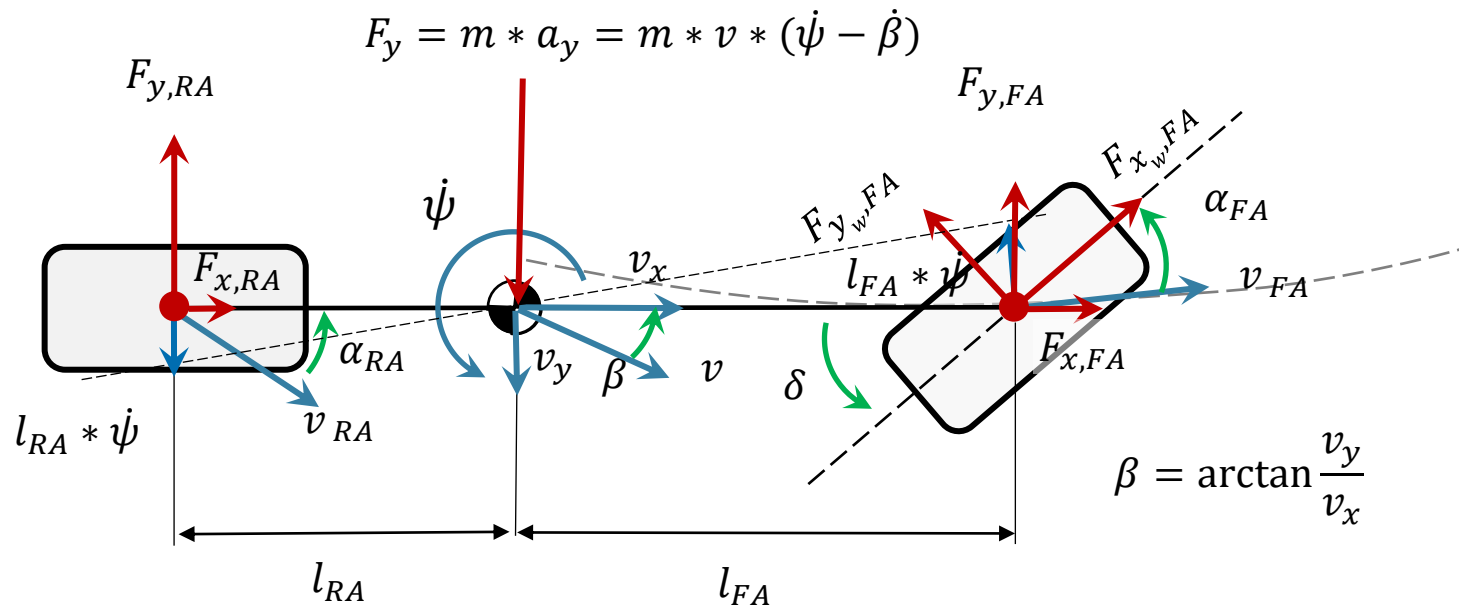
$$\sum F_y = 0$$

(2)

Longitudinal Motion

$$F_x = F_{x,FA} + F_{x,RA} = m * a_x$$

$$\sum F_x = 0$$

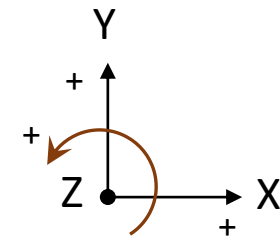


(3)

Rotation Z-Axis

$$\theta * \ddot{\psi} = F_{y,FA} * l_{FA} - F_{y,RA} * l_{RA}$$

$$\sum M_z = 0$$



Basic vehicle dynamics calculation and vehicle models

(1)

Lateral Motion

$$F_y = F_{y,FA} + F_{y,RA} = m * a_y = m * v * (\dot{\psi} - \dot{\beta})$$

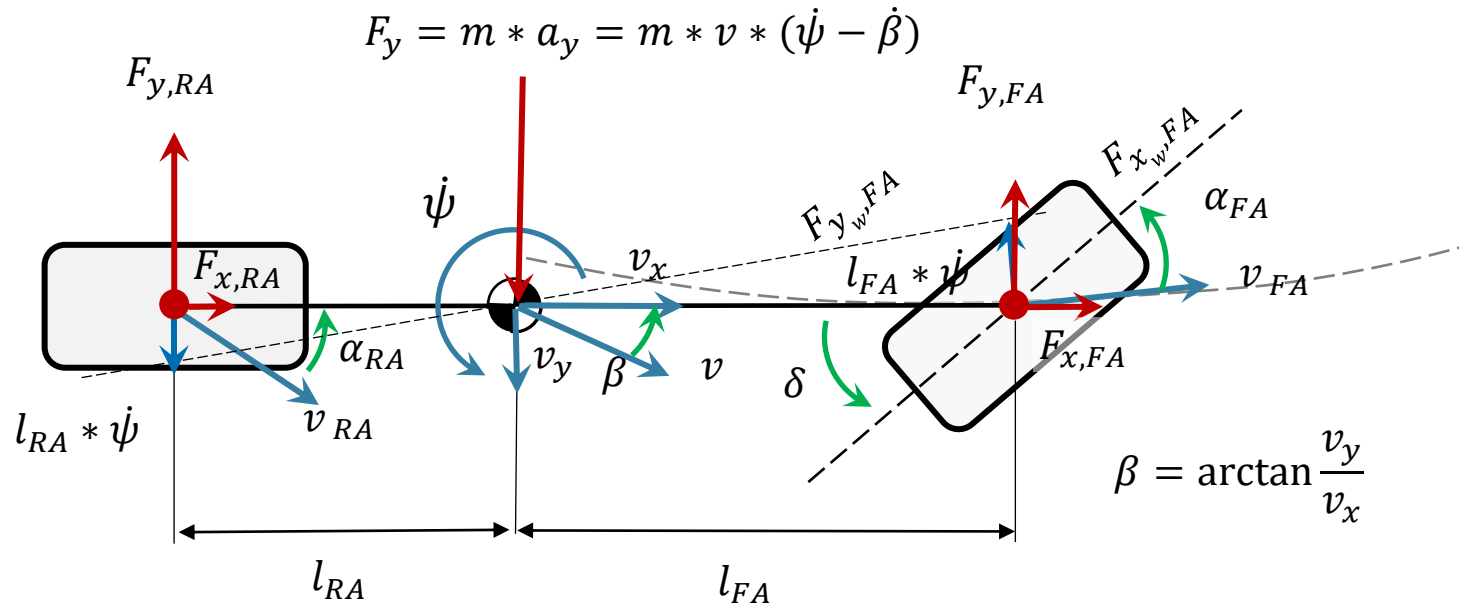
$$\sum F_y = 0$$

(2)

Longitudinal Motion

$$F_x = F_{x,FA} + F_{x,RA} = m * a_x$$

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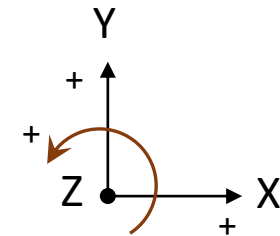


(3)

Rotation Z-Axis

$$\theta * \ddot{\psi} = F_{y,FA} * l_{FA} - F_{y,RA} * l_{RA}$$

$$\sum M_z = 0$$



Basic vehicle dynamics calculation and vehicle models

(1) Lateral Motion

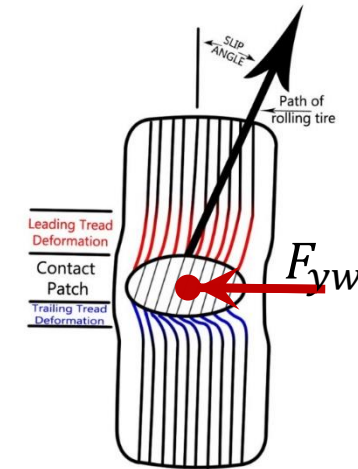
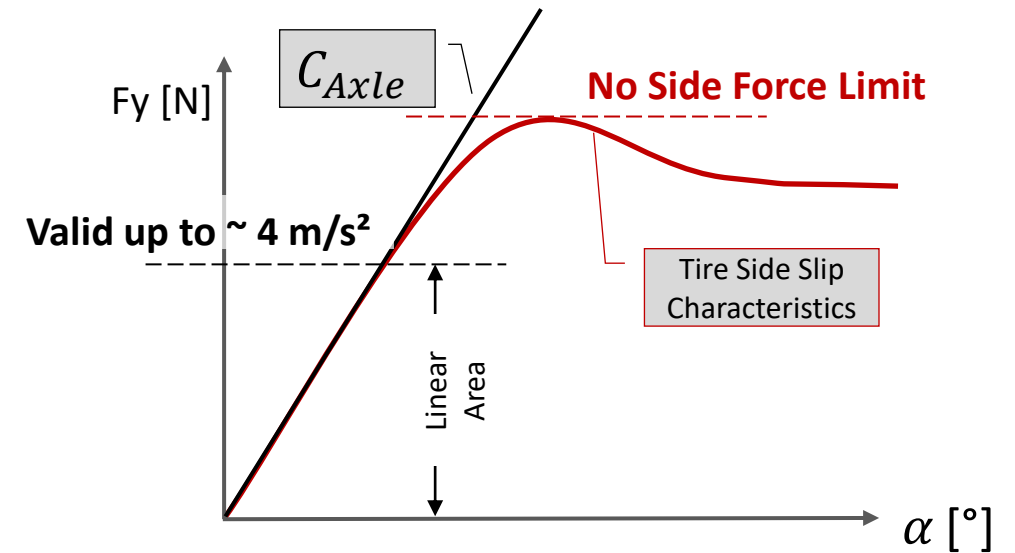
$$F_y = F_{y,FA} + F_{y,RA} = m * a_y = m * v * (\dot{\psi} - \dot{\beta})$$

$$(4) \quad F_{y,FA} = c_{FA} * \alpha_{FA}$$

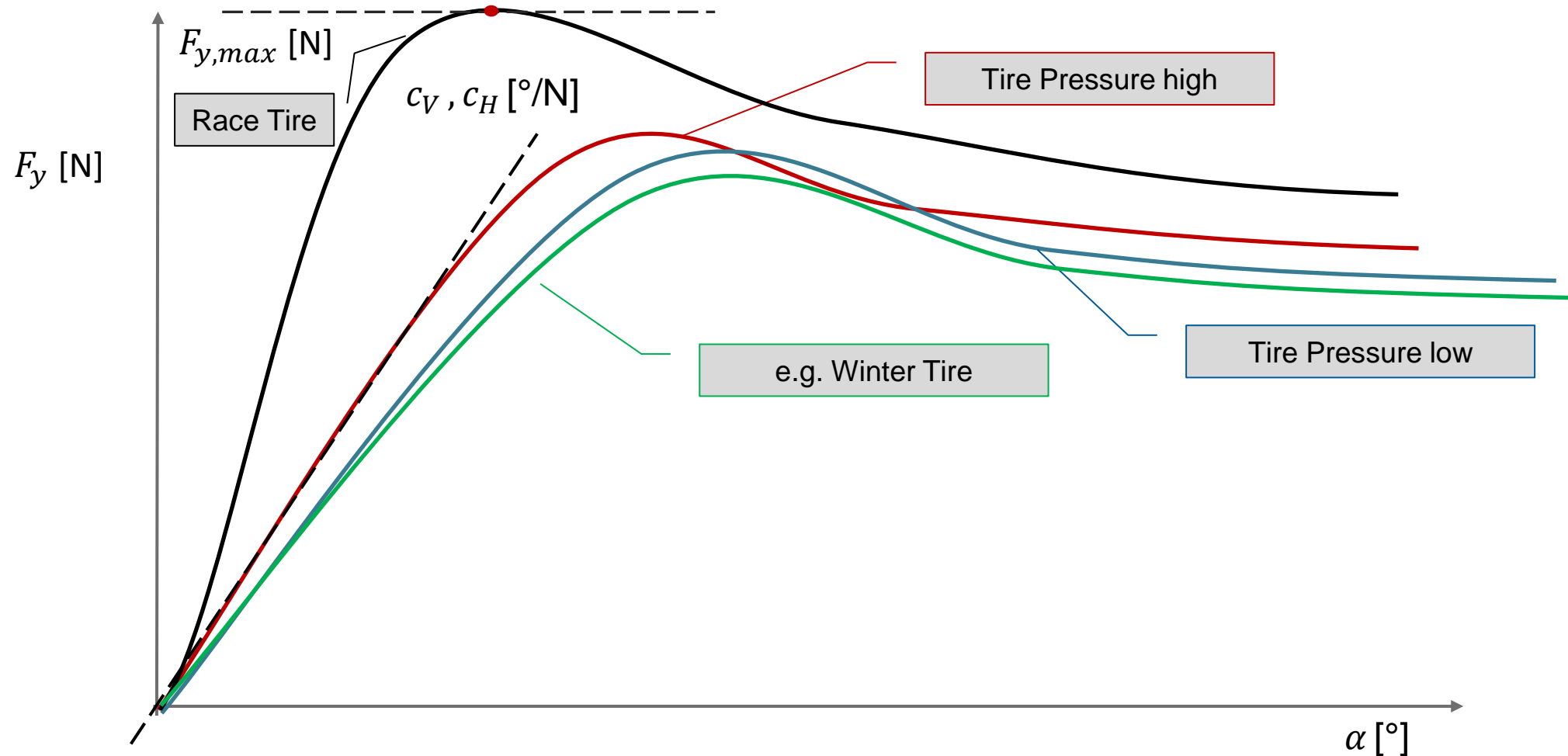
$$(5) \quad \alpha_{FA} = \delta + \beta - \frac{l_{FA} * \dot{\psi}}{v}$$

$$(4) \quad F_{y,RA} = c_{RA} * \alpha_{RA}$$

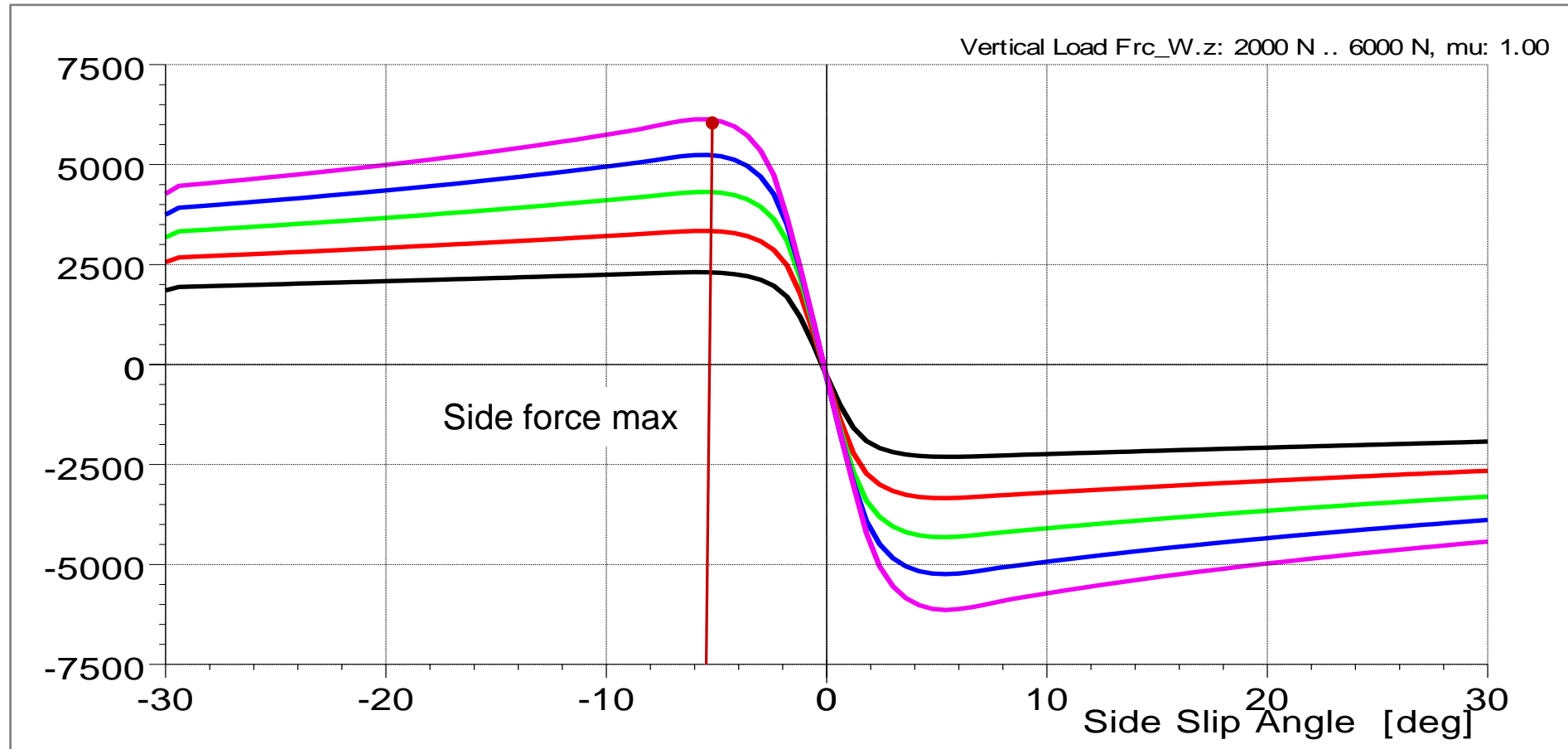
$$(5) \quad \alpha_{RA} = \beta + \frac{l_{RA} * \dot{\psi}}{v}$$



Tire characteristics for understeer behavior (qualitative)



Tire lateral characteristics: side slip behavior



Basic vehicle dynamics calculation and vehicle models

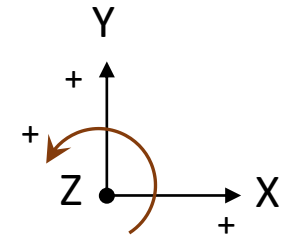
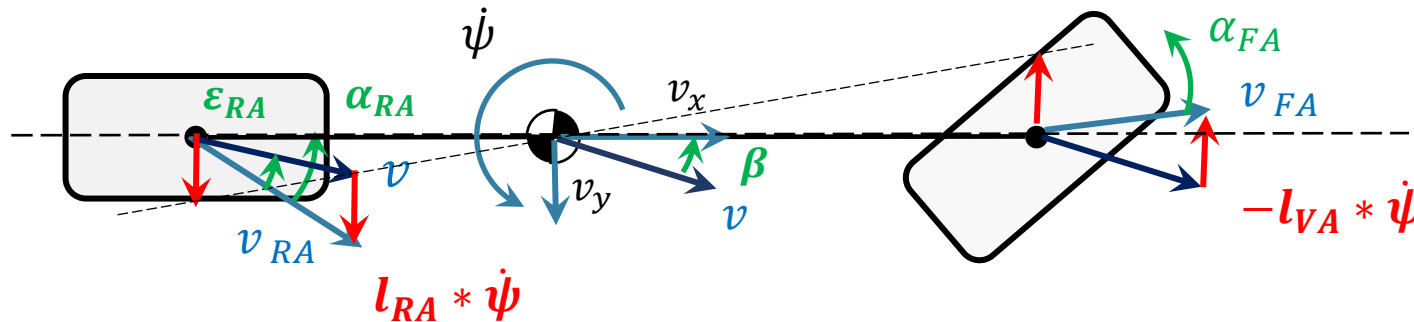
(5)
$$\alpha_{RA} = \beta + \frac{l_{RA} * \dot{\psi}}{v}$$

(5)
$$\alpha_{FA} = \delta + \beta - \frac{l_{FA} * \dot{\psi}}{v}$$

Small angle approximation

$$\epsilon_{RA} = \arctan \frac{l_{RA} * \dot{\psi}}{v} \sim \frac{l_{RA} * \dot{\psi}}{v}$$

$$\epsilon_{FA} = -\arctan \frac{l_{FA} * \dot{\psi}}{v} \sim -\frac{l_{FA} * \dot{\psi}}{v}$$



Basic vehicle dynamics calculation and vehicle models

$$(9) \quad c_{FA} * \alpha_{FA} = m * a_y * \frac{l_{RA}}{l}$$

$$(10) \quad c_{RA} * \alpha_{RA} = m * a_y * \frac{l_{FA}}{l}$$

with 5

with 5

$$(11) \quad c_{FA} \left(\delta + \beta - \frac{l_{FA} * \dot{\psi}}{v} \right) = m * a_y * \frac{l_{RA}}{l}$$

$$(12) \quad c_{RA} * \left(\beta + \frac{l_{RA} * \dot{\psi}}{v} \right) = m * a_y * \frac{l_{FA}}{l}$$

$$(13) \quad \beta = \frac{m}{l} * a_y * \frac{l_{RA}}{c_{FA}} + \frac{l_{FA} * \dot{\psi}}{v} - \delta$$

$$(14) \quad \beta = \frac{m}{l} * a_y * \frac{l_{FA}}{c_{RA}} - \frac{l_{RA} * \dot{\psi}}{v}$$

(15) equalize

$$\delta = \frac{m}{l} * a_y * \frac{l_{RA}}{c_{FA}} - \frac{m}{l} * a_y * \frac{l_{VA}}{c_{RA}} + \frac{l_{FA} * \dot{\psi}}{v} + \frac{l_{RA} * \dot{\psi}}{v}$$

Basic vehicle dynamics calculation and vehicle models

$$(16) \quad \delta = \frac{m}{l} * a_y * \frac{l_{RA}}{c_{FA}} - \frac{m}{l} * a_y * \frac{l_{VA}}{c_{RA}} + \frac{l_{FA} * \dot{\psi}}{v} + \frac{l_{RA} * \dot{\psi}}{v}$$

$$(17) \quad \delta = \frac{m}{l} * a_y * \left[\frac{l_{RA}}{c_{FA}} - \frac{l_{VA}}{c_{RA}} \right] + \frac{\dot{\psi}}{v} * (l_{FA} + l_{RA})$$

$$\delta = \frac{l}{r} + \frac{m}{l} * a_y * \left[\frac{l_{RA}}{c_{FA}} - \frac{l_{FA}}{c_{RA}} \right]$$

Steady-state cornering

$$(18) \quad \delta = \frac{l}{r} + \frac{m}{l} * a_y * \left[\frac{l_{RA}}{c_{FA}} - \frac{l_{FA}}{c_{RA}} \right]$$

Ackermann

Self-Steering Behavior

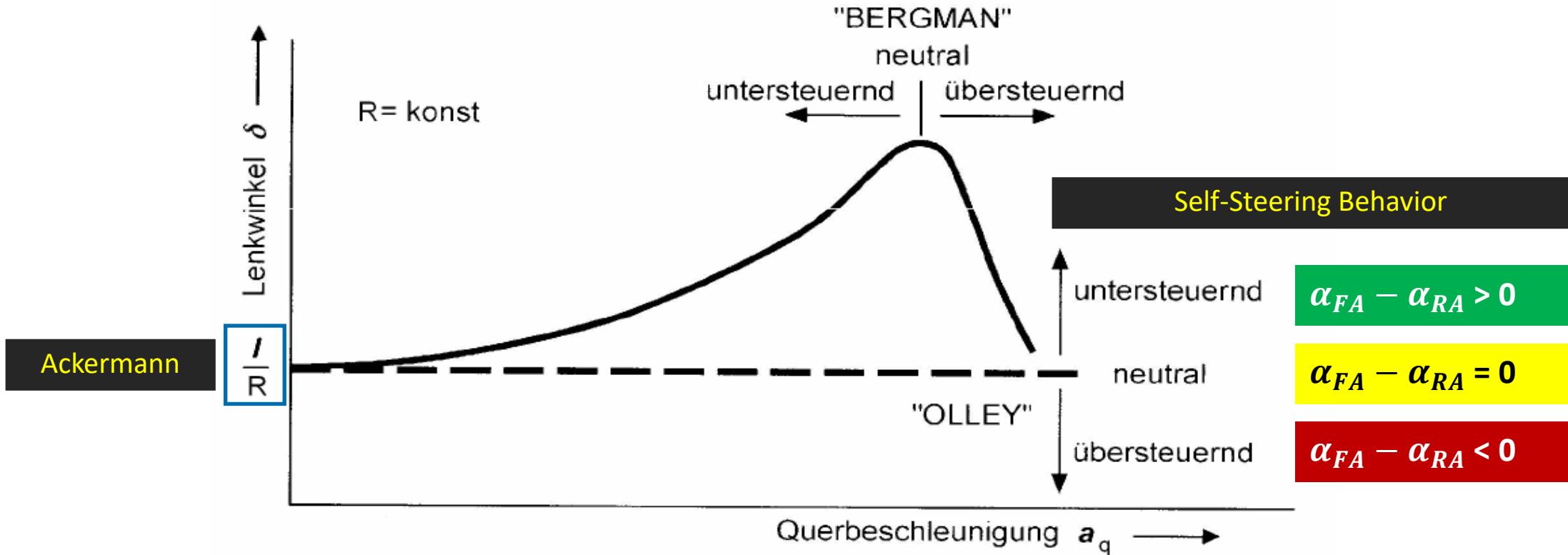
$$(19) \quad \delta = \frac{l}{r} + \Delta \alpha \quad (\Delta \alpha = \alpha_{FA} - \alpha_{RA})$$

Understeer and oversteer definition

Ackermann

Self-Steering Behavior

$$\delta = \frac{l}{r} + \Delta \alpha \quad (\Delta \alpha = \alpha_{FA} - \alpha_{RA})$$



Definition of Eigenlenkgradient (Understeer Gradient)

(20)

Calculation of Eigenlenkgradient

$$EG = \frac{1}{i_s} * \frac{d\delta_H}{da_y} - \frac{d\delta_A}{da_y}$$

= 0 if $R = \text{const}$ since $d\delta_A \approx \frac{l}{R}$

Calculation of **the EG** = Eigenlenkgradient
(engl. understeer gradient)
 $d\delta_H$: Steering wheel angle
 $d\delta_A$: Ackermann angle

(21)

Calculation of specific Eigenlenkgradient

$$EG_H = \frac{d\delta_H}{da_y} - \frac{d\delta_A}{da_y}$$

H = Hand at Steering Wheel

Definition of Eigenlenkgradient (Understeer Gradient)

(20)

Calculation of Eigenlenkgradient

$$EG = \frac{1}{i_s} * \frac{d\delta_H}{da_y} - \frac{d\delta_A}{da_y}$$

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Calculation of **the EG** = Eigenlenkgradient
(engl. understeer gradient)
 δ_H : Steering wheel angle
 δ_A : Ackermann angle

(21)

Calculation of specific Eigenlenkgradient

$$EG_H = \frac{d\delta_H}{da_y} - \frac{d\delta_A}{da_y}$$

H = Hand at Steering Wheel

$EG = 0$: Neutral

$EG < 0$: Oversteer

$EG > 0$: Understeer

Characteristic Velocity and Critical Velocity

(22)

Calculation of Characteristic Velocity

$$\left[\frac{\dot{\psi}}{\delta_H} \right]_{v_{char}} = \frac{1}{2} * \left[\frac{\dot{\psi}}{\delta_H} \right]_{EG=0}$$

if $\frac{\dot{\psi}}{\delta_H} > \left[\frac{\dot{\psi}}{\delta_H} \right]_{EG=0}$ and $R = const$

i.e. if vehicle shows steady-state understeer

(23)

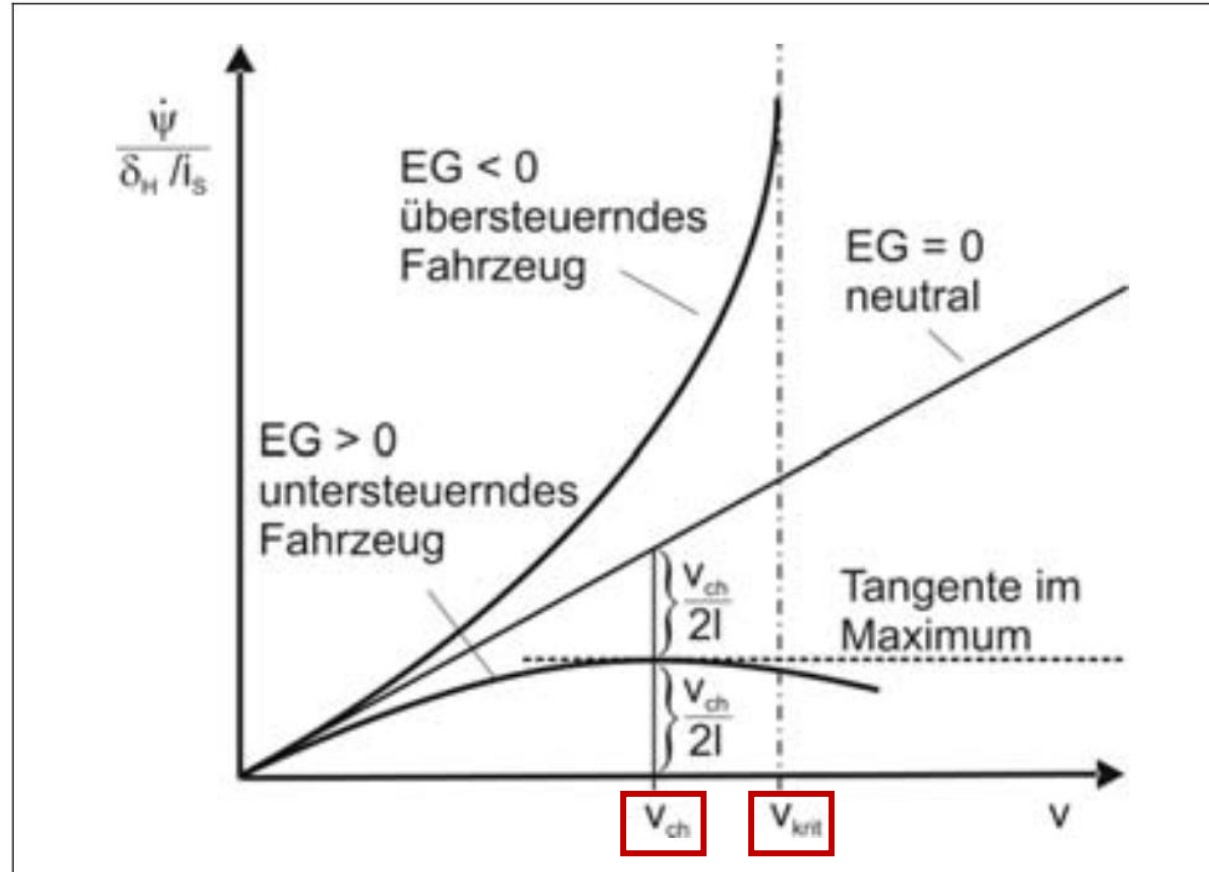
Calculation of Critical Velocity

$$\left[\frac{\dot{\psi}}{\delta_H} \right]_{v_{crit}} = \frac{1}{2} * \left[\frac{\dot{\psi}}{\delta_H} \right]_{EG=0}$$

if $\frac{\dot{\psi}}{\delta_H} < \left[\frac{\dot{\psi}}{\delta_H} \right]_{EG=0}$ and $R = const$

i.e. if vehicle shows steady-state oversteer

Characteristic Velocity and Critical Velocity



[4]

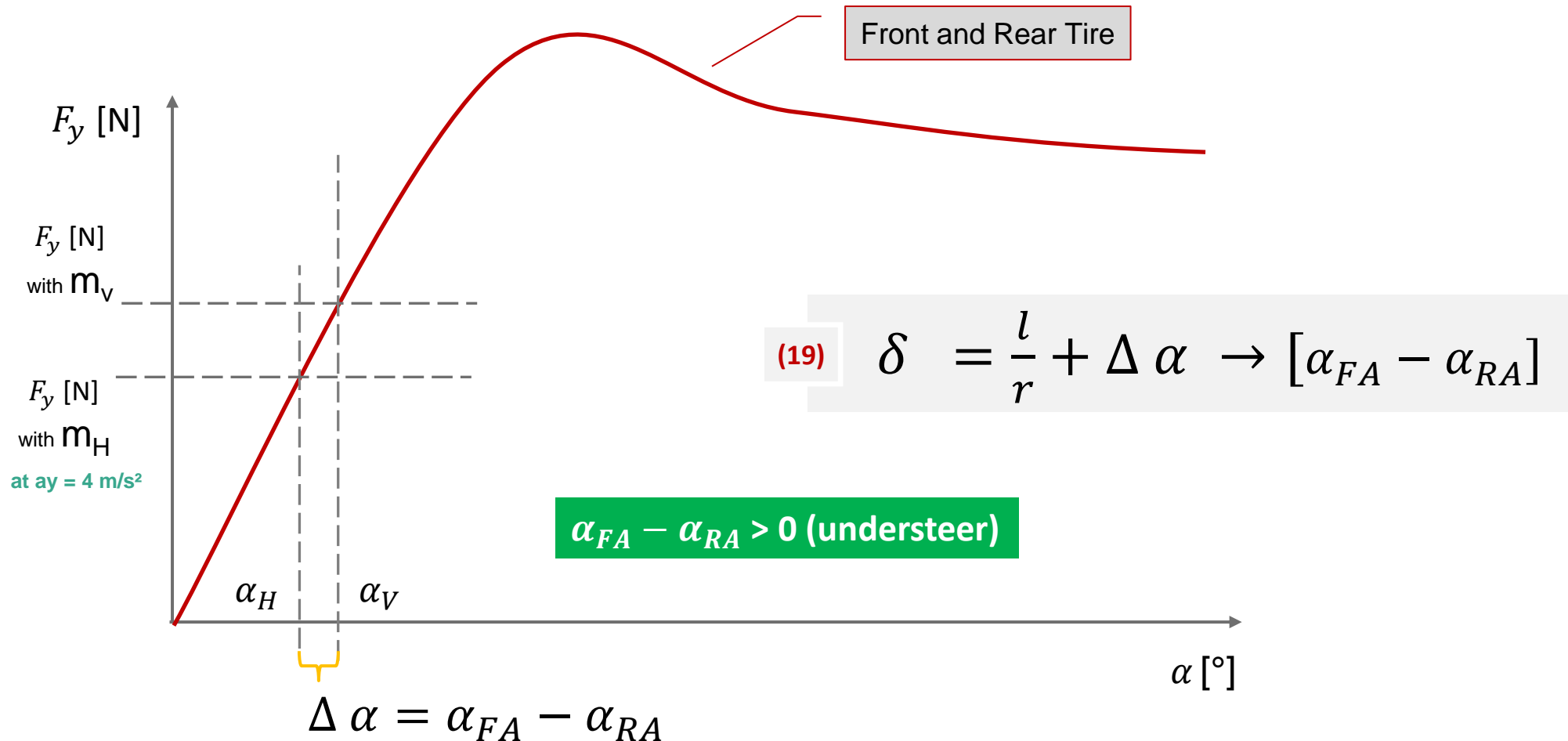
Practice Session: 15 min

- You have a vehicle with the following data:
 - Mass $m = 1600$ kg
 - Wheelbase = 2540 mm
 - Track width = 1420 mm
 - $m_{FA} = 880$ kg
 - Steering ratio = 1/15
 - Yaw inertia $J_z = 2800$ kgm²
 - Cornering stiffness = 3000 N/°
- Calculate the following:
 1. CoG in x-direction
 2. Ackermann steering angle for constant cornering at $R = 100$ m
 3. EG between 0 and 4 m/s²

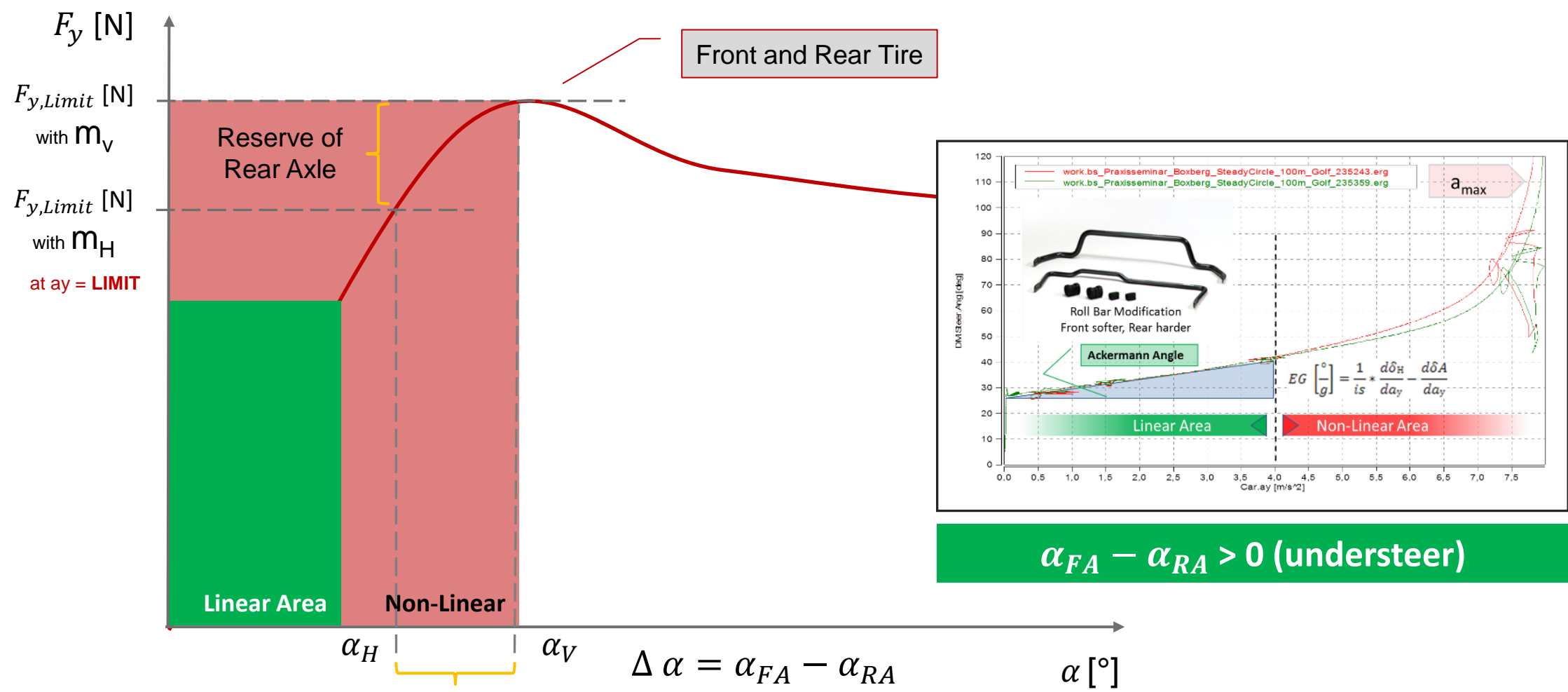
Practice Session: 15 min

- How much steering wheel angle is necessary if your driver wants to corner at a steady acceleration of 4 m/s^2 ?
- Which three measures do you recommend to tune the vehicle towards less understeer (assuming only knowledge of the single track model)?
- How would you achieve an increase in understeering gradient by 50%?

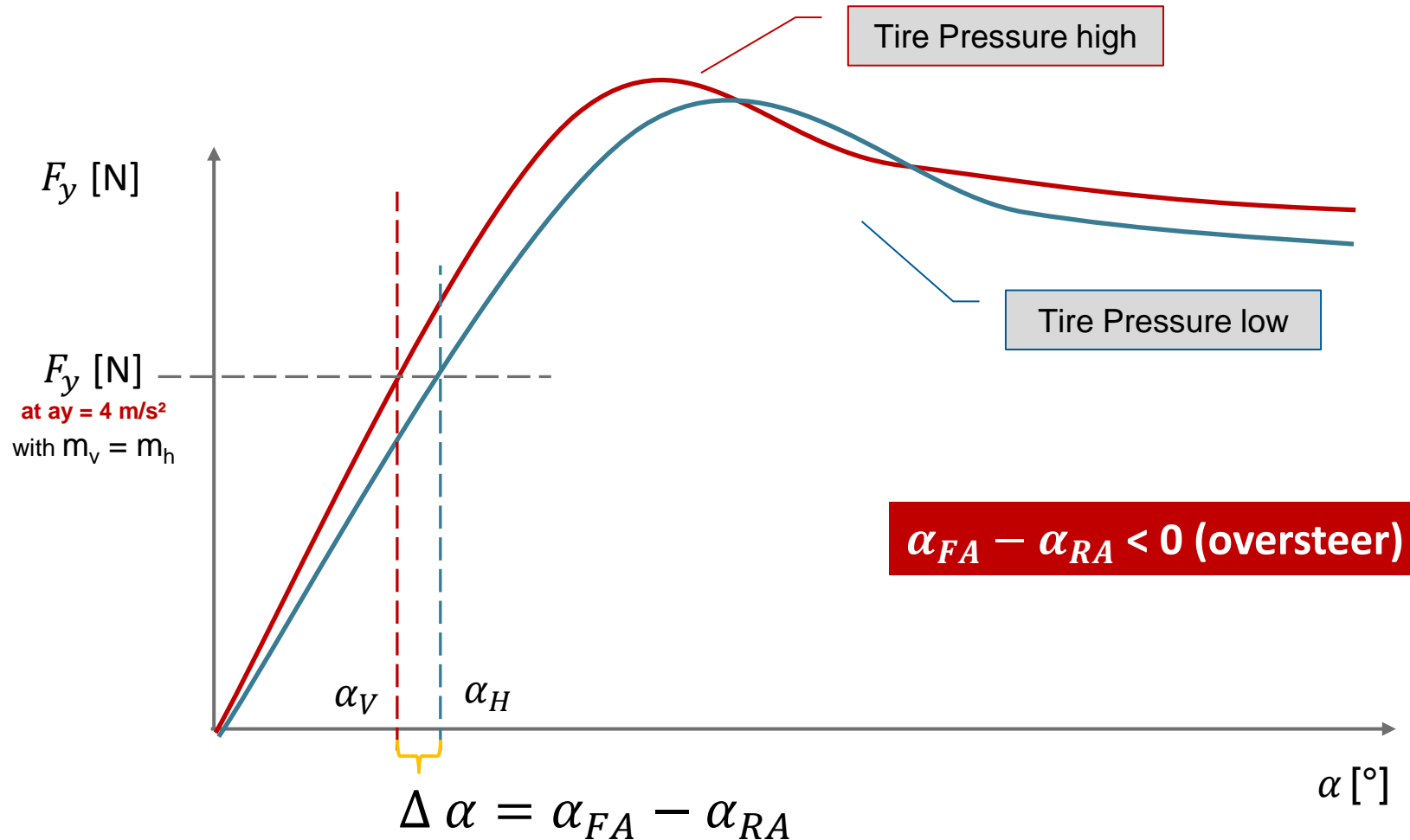
Case 1: Understeer behavior with wheel load distribution



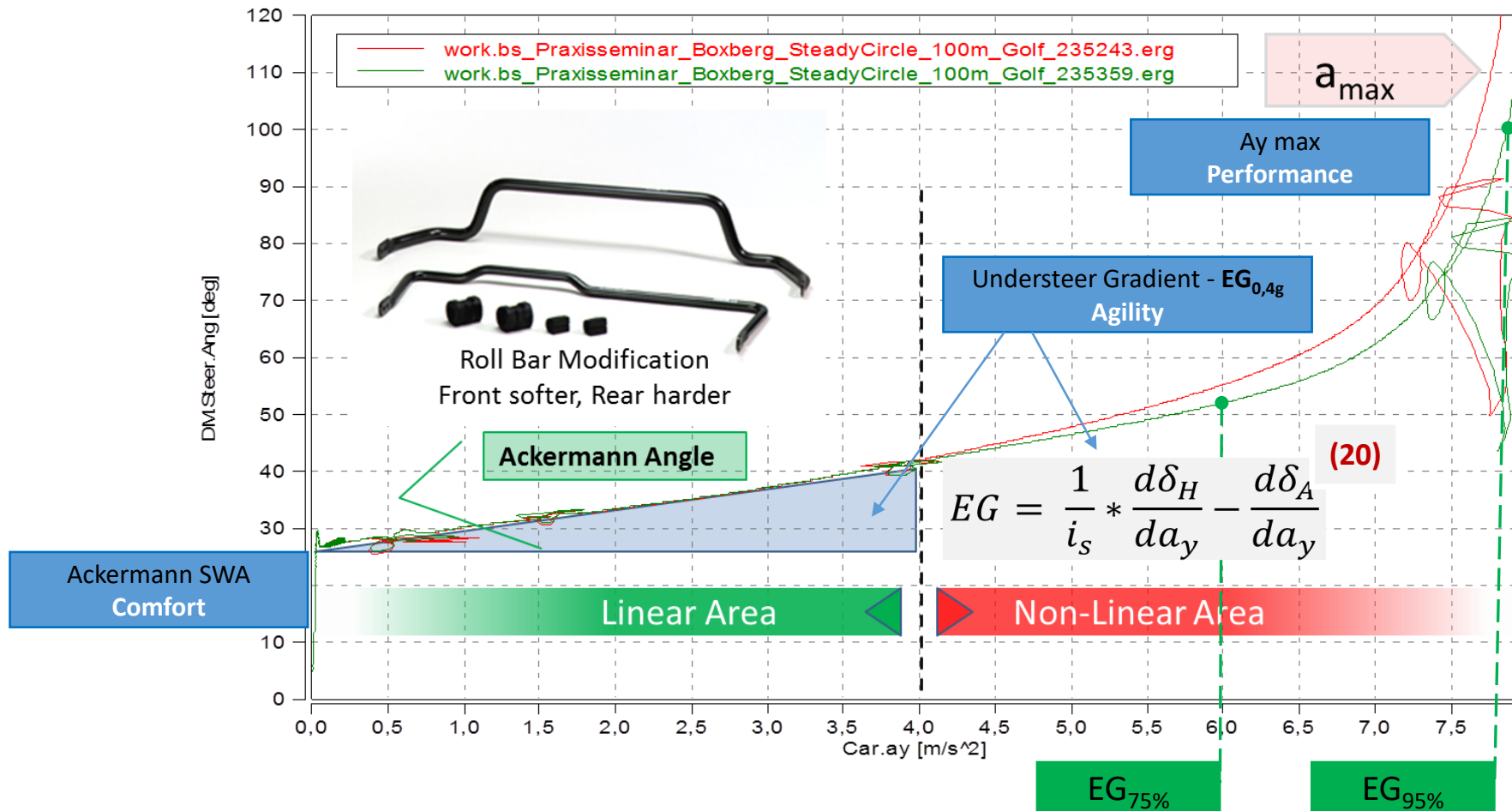
Case 1: Understeer behavior with wheel load distribution



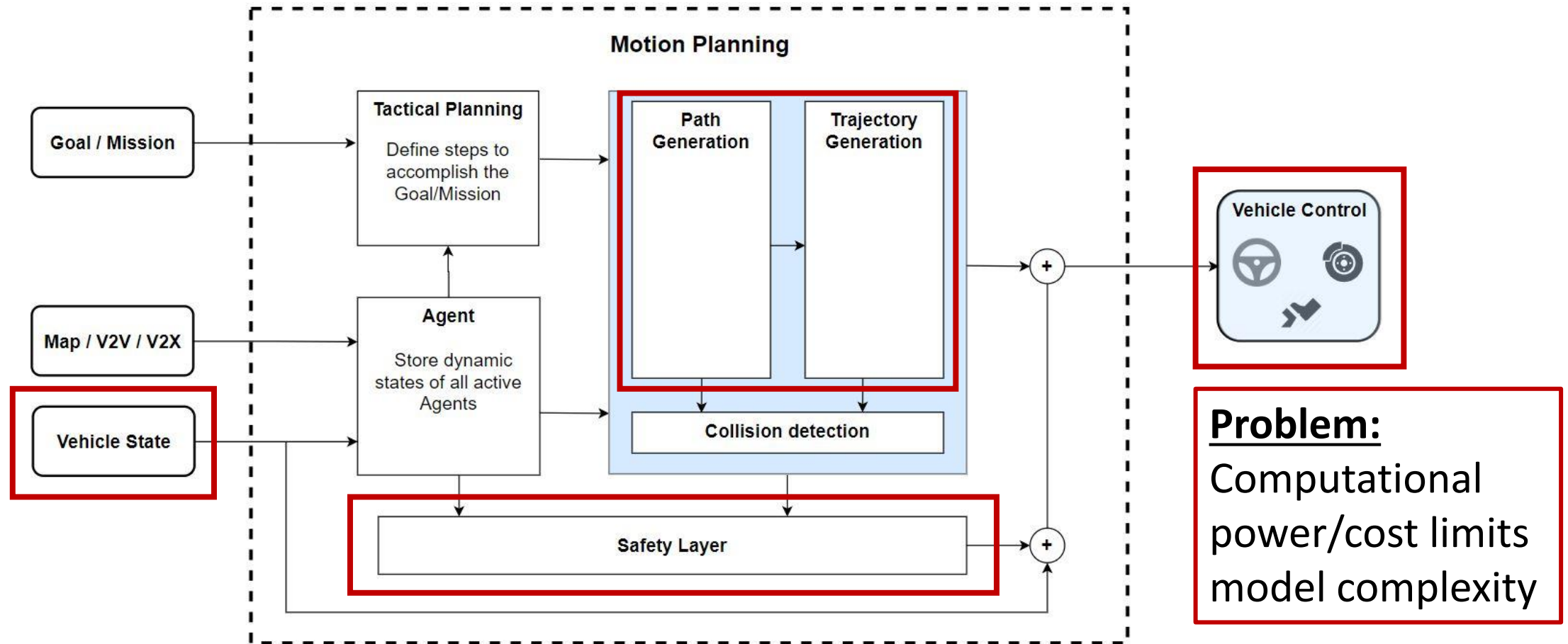
Case 2: Oversteer behavior with tire pressure difference



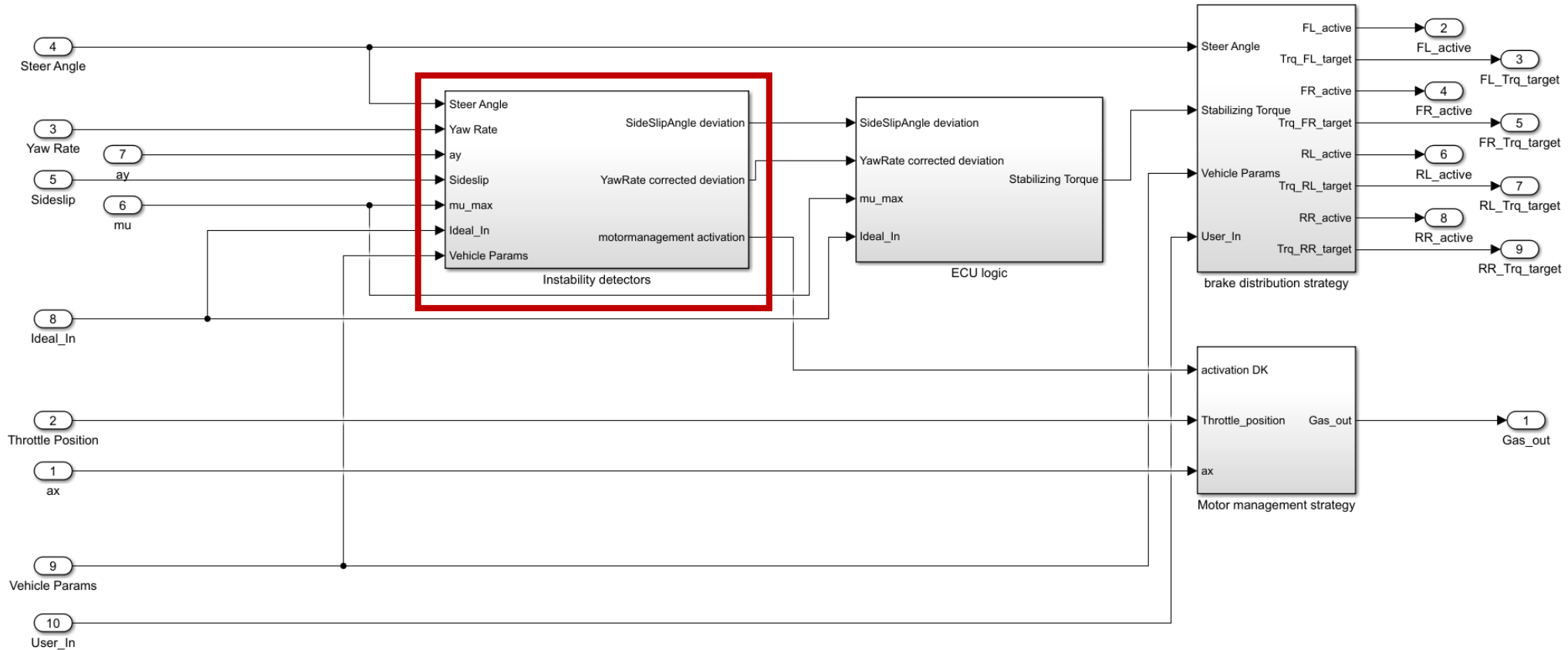
Model behavior in steady-state cornering



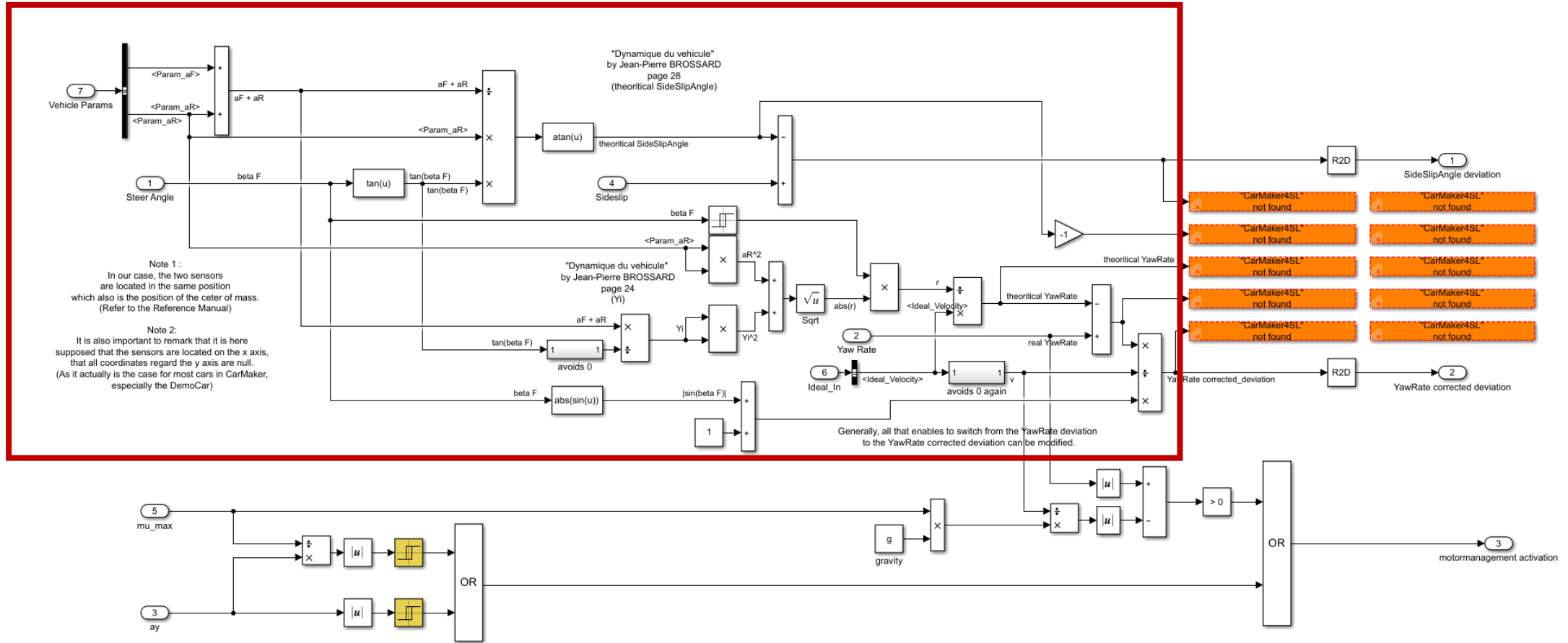
Use-Cases for a Single-track model – Model-based motion planning



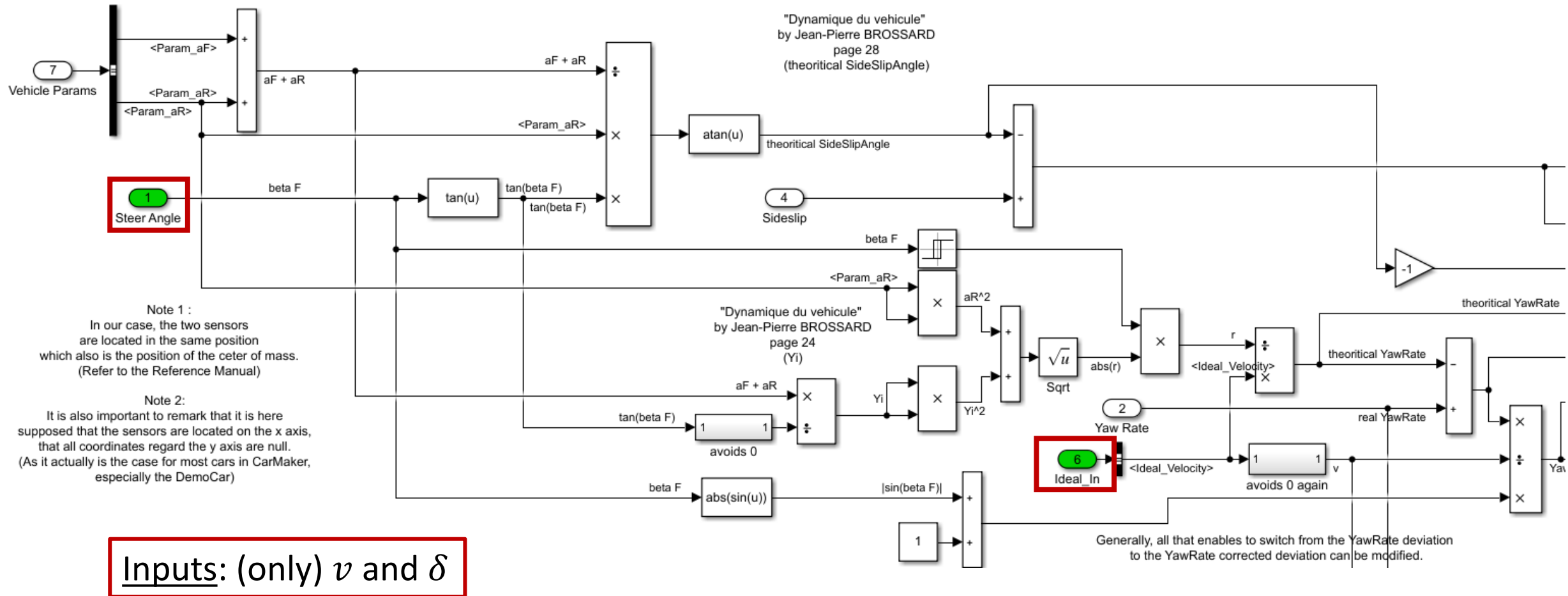
Use-Cases for a Single-track model – Model-based control



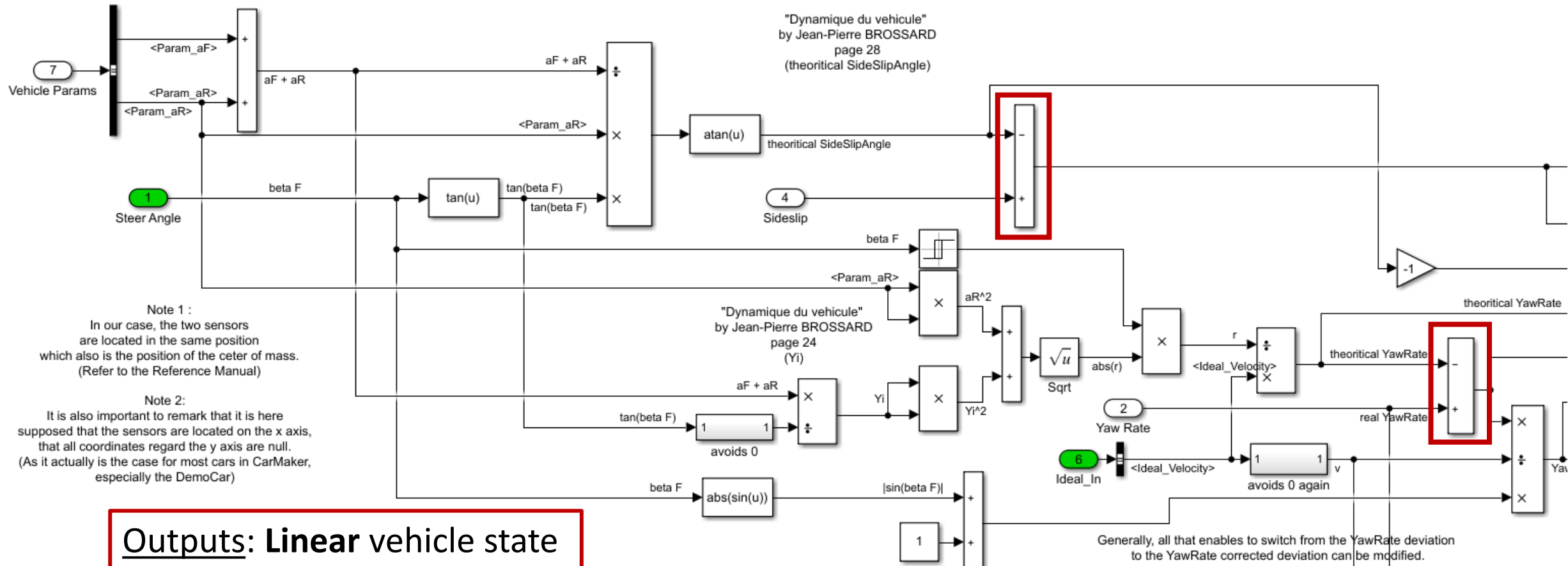
Use-Cases for a Single-track model – Model-based control



Use-Cases for a Single-track model – Model-based control




Use-Cases for a Single-track model – Model-based control



Model-based control – CarMaker example

CarMaker - Test: Examples/VehicleDynamics/StabilityControl/ESC_SineWithDwell - 'Il-nb-30' online

File Application Simulation Parameters Settings Help



Car: Examples/DemoCar
Typical, unvalidated data for passenger car with Front Wheel Drive

Trailer: -

Tires: Ex.../RT_195_65R15 Ex.../RT_195_65R15

Ex.../RT_195_65R15 Ex.../RT_195_65R15

Load: 70.0 kg
70.0 kg

Maneuver

0	40.0	80		
1	1.0714	GBCP	lat:Sinus	
2	0.5	GBCP	lat±+0	
3	0.357	GBCP	lat±Sinus	
4	5	GBCP	lat:0	

Simulation

Perf.: max
Status: (26.4×)
Idle

Time: 46.9
Distance: 822.07

Storage of Results

Mode: collect only

Buffer: 134.2 MB, 29 s

Start
Stop

Save Stop Abort

CarMaker - Vehicle Data Set: Examples/DemoCar

Vehicle Data Set

Vehicle Body Bodies Engine Mount Suspensions Steering Tires Brake Powertrain Aerodynamics Sensors

Brake Model: Hydraulic

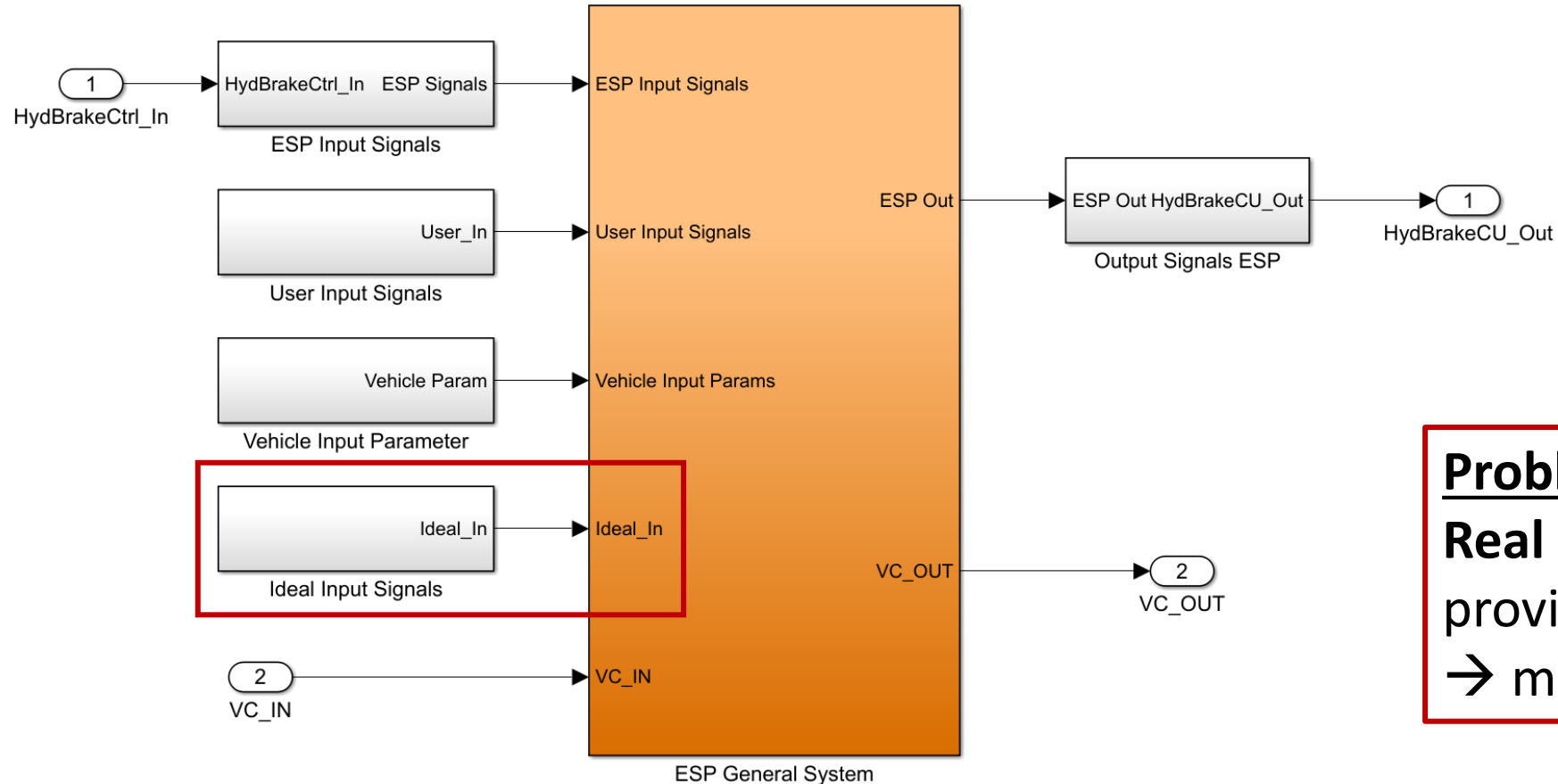
General Control System

Model: ESP RTW

Modelclass-specific Parameters

For FMU please use FMU Plug-ins.

Use-Cases for a Single-track model – Vehicle state estimation



Problem:
Real vehicles don't
provide **ideal** in-/outputs
→ model-based estimate

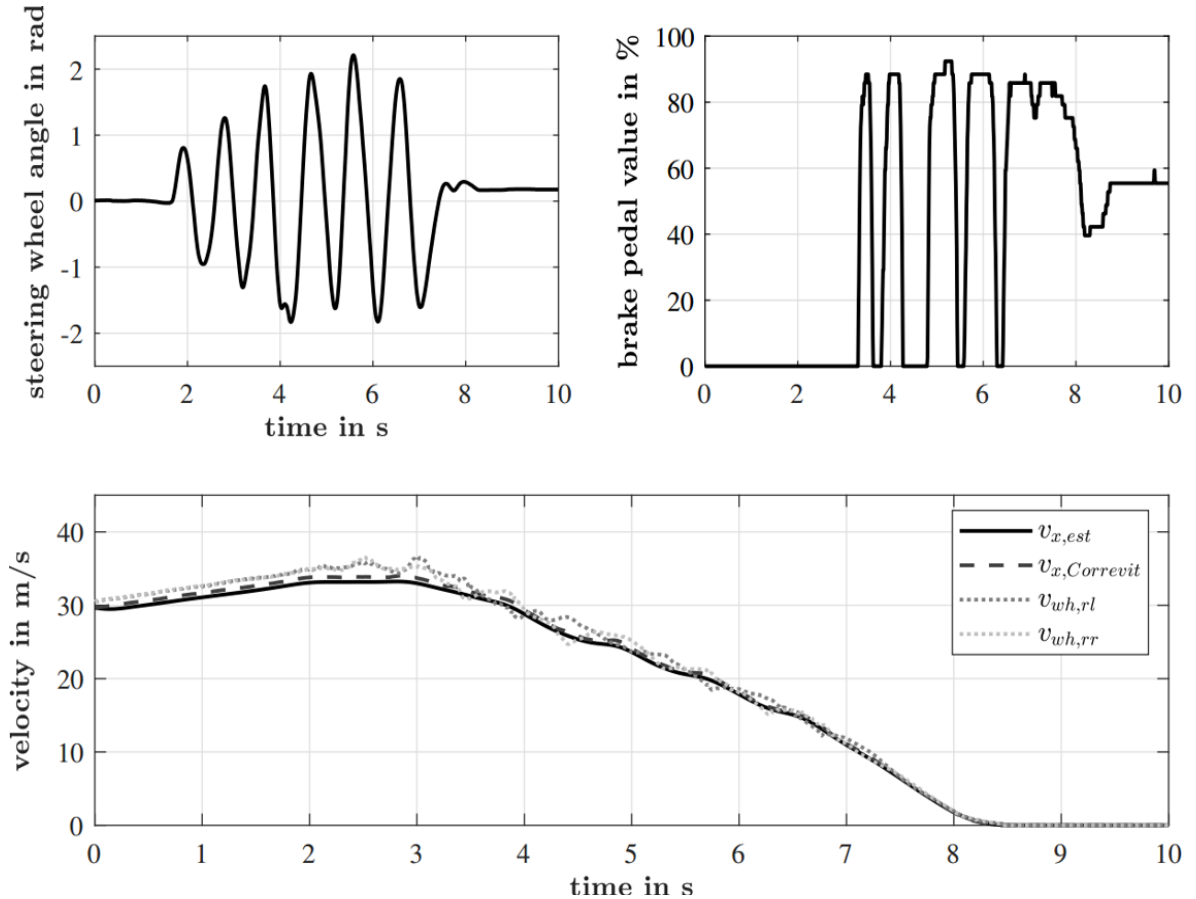
Use-Cases for a Single-track model – Vehicle state estimation

Estimation of non-measurable quantities

- **Longitudinal velocity** under slip
- **Lateral velocity**
- **Tire forces**
- Available **friction coefficient** (tire potential)
- Road bank angle
- ...

Prediction of vehicle state

- Motion planning
- Advanced control



What can we improve? – Extensions of the Single-Track model

Two-track model (6 body-DOF)

- Consider **roll**, **heave** and **pitch** motion
 - **CoG height** is now relevant
 - Four wheels with **dynamic wheel load transfer**

Non-linear tire modeling

- Saturation through long./lat. force limits (tire potential)
- Steering angle on both axles
- Slip-angle contribution of resulting axle stiffness
- Transient vertical dynamics
- Combined slip modeling
- ...

→ How complex is too complicated?

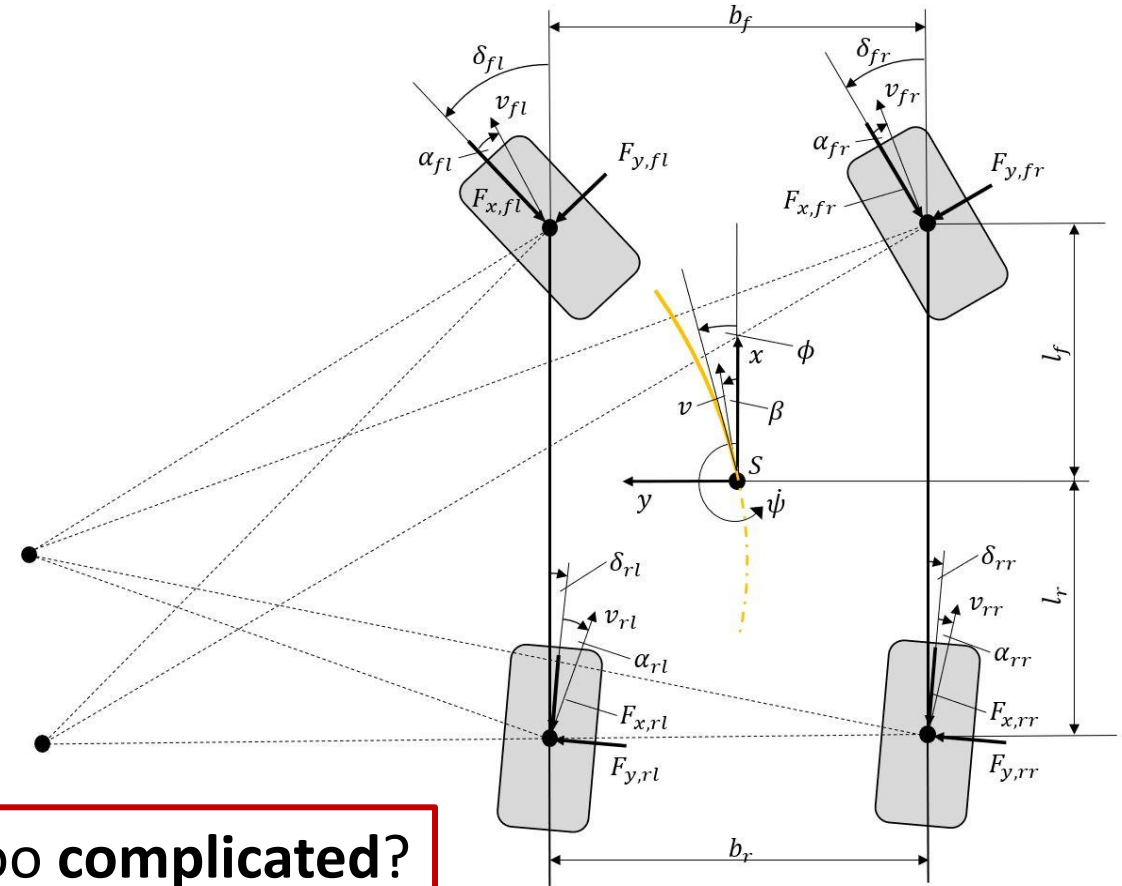
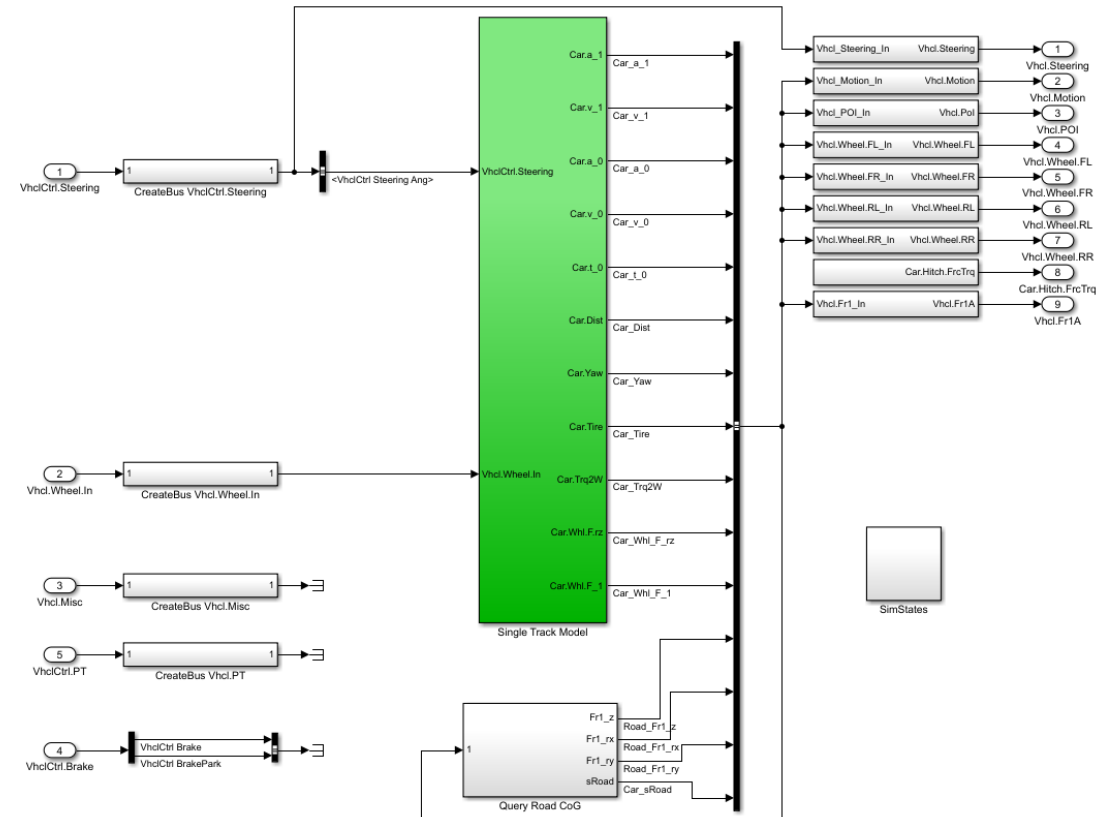
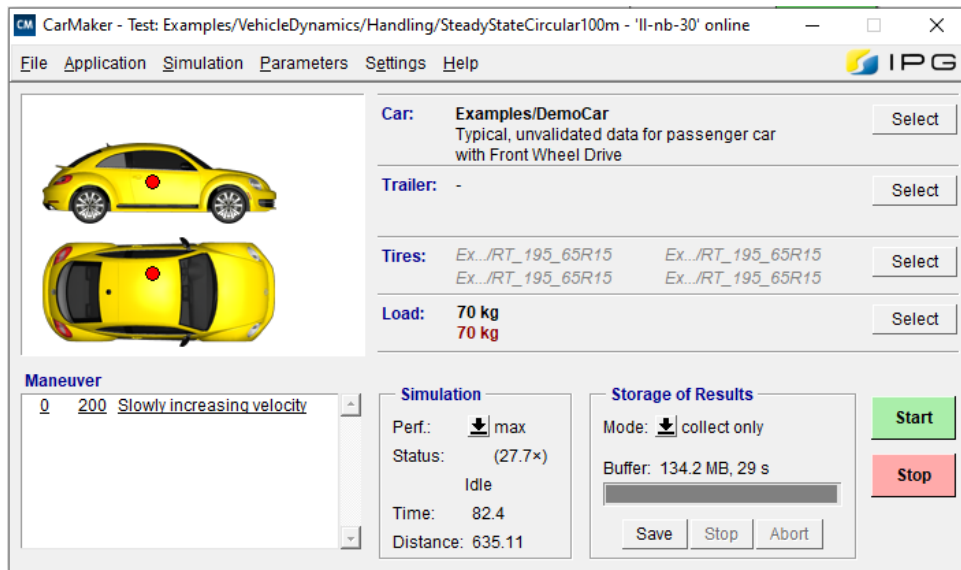


Bild durch Klicken auf Symbol hinzufügen

Validity of a single-track model: CarMaker exercise



Validity of the simplified model

CarMaker - Vehicle Data Set Generator

Vehicle Data Set Generator [Generate] [Close]

Basic Settings | Advanced Settings

Generate

☒ Vehicle Data Set Compact.car

☒ Vehicle Graphics Compact.tcl

Vehicle Class

☐ Small Car

☒ Compact Car

☐ Medium Car

☐ Luxury Car

☐ Delivery Van

☐ Compact SUV

☐ Full Size SUV

Vehicle Parameters

Unloaded weight 1320.0 kg

Vehicle length 4350.0 mm

Vehicle width 1790.0 mm

Vehicle height 1455.0 mm

Wheel base 2635.0 mm

Track width front 1551.0 mm

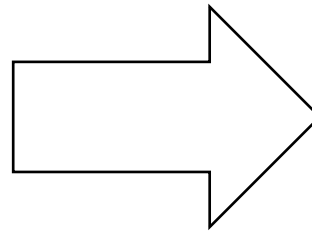
Track width rear 1549.0 mm

Rear overhang 845.0 mm

Tire

Tire size 205 / 55 R 16

☒ Generate Tire Data Set [Tire Data Generator]



CarMaker - Vehicle Data Set: SingleTrack_RTW_mod

Vehicle Data Set [File] [Close]

Assembly | **Body** | Suspensions | Steering | Tires | Brake | Powertrain | Sensors | Vehicle Control | Additional

Structure

Outer Shell

Aero-dynamics

Vehicle Body: SingleTrack RTW [Edit]

☐ Manual body split

	Position x / y / z [m]			Mass [kg]	M. of inertia x / y / z [kgm²]		
Body	2.7	0	0.31	650	1000	2000	3000
Body B	2.43	0.0	0.6	650.5	235.0	750.0	800.0
Joint A - B	2.43	0.0	0.6				

Extra Vehicle Model Parameters [Folder Icon]

Stiffness

Mode: Characteristic Value

Stiffness [Nm/deg]	Rotation X (Torsion)		Rotation Y (Bending)	
	Angle [deg]	Torque [Nm]	Angle [deg]	Torque [Nm]
5000.0	0.0	0.0	0.0	0.0
	0.5	2500.0	0.5	7500.0
	1.0	5000.0	1.0	15000.0

Amplification [-] 1.0

Damping [Nms/deg] 100.0

Damping [Nms/deg] 100.0

Amplification [-] 1.0

Joint Body A - Body B

Validity of the simplified model

- Generate a data set of your choice
 - Group 1: Compact
 - Group 2: Medium
 - Group 3: Luxury
 - Group 4: Compact SUV
 - Group 5: Full-size SUV

The screenshot shows the 'CarMaker - Vehicle Data Set Generator' window. It has a title bar with 'CM' and standard window controls. The main title is 'Vehicle Data Set Generator' with 'Generate' and 'Close' buttons. There are two tabs: 'Basic Settings' (selected) and 'Advanced Settings'. Under 'Basic Settings', there is a 'Generate' section with two checked items: 'Vehicle Data Set' (with a text field containing 'Compact.car') and 'Vehicle Graphics' (with a text field containing 'Compact.tcl'). Below this is the 'Vehicle Class' section with radio buttons for 'Small Car', 'Compact Car' (selected), 'Medium Car', 'Luxury Car', 'Delivery Van', 'Compact SUV', and 'Full Size SUV'. To the right is the 'Vehicle Parameters' section with a table of values: Unloaded weight (1320.0 kg), Vehicle length (4350.0 mm), Vehicle width (1790.0 mm), Vehicle height (1455.0 mm), Wheel base (2635.0 mm), Track width front (1551.0 mm), Track width rear (1549.0 mm), and Rear overhang (845.0 mm). At the bottom is the 'Tire' section with a 'Tire size' field showing '205 / 55 R 16' and a checked 'Generate Tire Data Set' option with a 'Tire Data Generator' button.

Vehicle Parameters	
Unloaded weight	1320.0 kg
Vehicle length	4350.0 mm
Vehicle width	1790.0 mm
Vehicle height	1455.0 mm
Wheel base	2635.0 mm
Track width front	1551.0 mm
Track width rear	1549.0 mm
Rear overhang	845.0 mm

Validity of the simplified model

- Calculate the maximum (vertical) wheel load transfer for steady state cornering at $a_y = 10 \text{ m/s}^2$
(hint: $F_{y,\text{cog}} = a_y * m$) for your own generated vehicle data set.
- Simulate the maneuver and compare the resulting wheel load difference between inner and outer wheels

→ How would you explain the different results to the CarMaker wheel loads?

CarMaker - Vehicle Data Set Generator

Vehicle Data Set Generator [Generate] [Close]

Basic Settings | Advanced Settings

Generate

☒ Vehicle Data Set Compact.car

☒ Vehicle Graphics Compact.tcl

Vehicle Class

☐ Small Car

☒ Compact Car

☐ Medium Car

☐ Luxury Car

☐ Delivery Van

☐ Compact SUV

☐ Full Size SUV

Vehicle Parameters

Unloaded weight	1320.0	kg
Vehicle length	4350.0	mm
Vehicle width	1790.0	mm
Vehicle height	1455.0	mm
Wheel base	2635.0	mm
Track width front	1551.0	mm
Track width rear	1549.0	mm
Rear overhang	845.0	mm

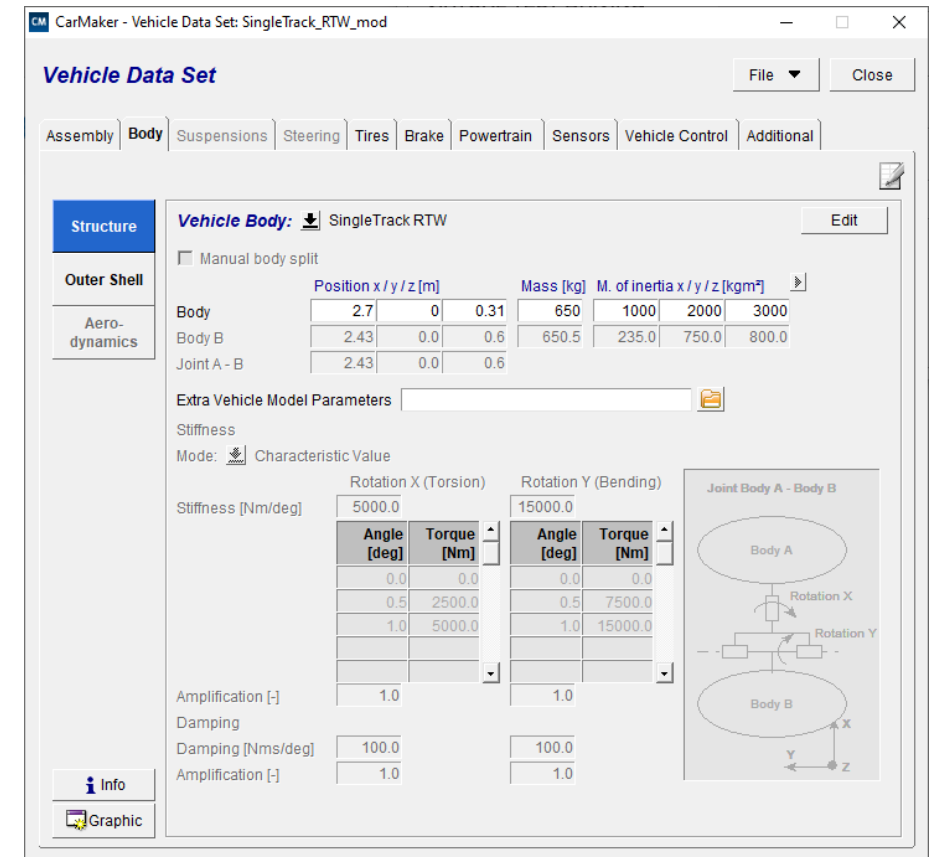
Tire

Tire size 205 / 55 R 16

☒ Generate Tire Data Set [Tire Data Generator]

Validity of the simplified model

- Fit a single-track model (SingleTrack_RTW_mod) to your steady-state model behaviour (hint: steady-state cornering, a_y vs. SWA, as before)



Validity of the simplified model

- Calculate the mean EG between standstill and 10 m/s^2 in increments of 2 m/s^2 (zero to two, zero to four...) for both the single-track model and your own vehicle model (hint: MXeval)
- Does the single-track model show sufficient accuracy (max. 10% deviation)?
 - If so, until which lateral acceleration?
 - Please explain the different results

